Maths First Aid Kit

Preparation for Engineering for Designers / Mechanisms and Structures

For all PDT 1 and CPD 2 students,

Next semester you will be studying either Mechanisms and Structure (PDT1) or Engineering for Designers (CPD2), with me Drew Batchelor. One of key learning outcomes for these modules is to ensure that all UWE product Design Graduates have sufficient numeracy to work as professional designers.

If you don't feel comfortable with GCSE maths or haven't studied either AS-Level or A-Level maths or physics. The attached maths first aid kit will help you bring those rusty maths skills up to speed. We understand that some students are not confident with numeracy, you now have an opportunity to fix that, as it is an essential competency for the technical aspects of being a professional designer.

This handout is (mostly) revision of GCSE material, It is the foundation with underpins all the analysis in the module. By the first seminar (week of 16th January 2012) you're expected to ensure that you are comfortable with all of the material in this handout. Tutors will provide support, but for this to work, you will need to engage with your own learning, (please do*, it will save you a lot of hassle next summer).

How to use this handout

Starting this week, work through this handout, working in pairs or small groups will help. Try the exercises at the end of each topic, if you can do them, move on to the next sheet. If you find it difficult, work through the whole topic, if you still have some questions on that topic please either; visit Espressomaths or visit Drew during his clinic hours.

The last 3 topics (2.10/2.11/2.12) are the most important. When you can do those, you will know that you will be comfortable with the level of maths on this module.

Topics Covered in this handout

During the first week of tutorials we will also cover the following: SI Units, Significant figures – Precision and accuracy, Subscripts, Pythagoras & Trigonometry, Using Scientific Calculator.

Espressomaths Support

http://www.cems.uwe.ac.uk/espressomaths/

UWE provides free 1-to-1 maths tutoring every day – if you need it, please use it, starting this week – take this handout along. Espressomaths is available 1200-1400, Monday-Friday in the corner of OneZone Refectory near Core24 in E-block.

Espressomaths has an associated handout repository, situated in Room 2Q53, the handouts are also available online: http://www.cems.uwe.ac.uk/espressomaths/resource_centre.html
The site also provide links to other online excellent learning resources:
http://www.cems.uwe.ac.uk/espressomaths/support_materials.html#maths

Good luck,
Drew

(*pretty please.)

Engineering Maths First Aid Kit

1.1

Fractions

Introduction

The ability to work confidently with fractions, both number fractions and algebraic fractions, is an essential skill which underpins all other algebraic processes. In this leaflet we remind you of how number fractions are simplified, added, subtracted, multiplied and divided.

1. Expressing a fraction in its simplest form

In any **fraction** $\frac{p}{q}$, say, the number p at the top is called the **numerator**. The number q at the bottom is called the **denominator**. The number q must never be zero. A fraction can always be expressed in different, yet **equivalent** forms. For example, the two fractions $\frac{2}{6}$ and $\frac{1}{3}$ are equivalent. They represent the same value. A fraction is expressed in its **simplest form** by cancelling any factors which are common to both the numerator and the denominator. You need to remember that factors are numbers which are multiplied together. We note that

$$\frac{2}{6} = \frac{1 \times 2}{2 \times 3}$$

and so there is a factor of 2 which is common to both the numerator and the denominator. This common factor can be cancelled to leave the equivalent fraction $\frac{1}{3}$. Cancelling is equivalent to dividing the top and the bottom by the common factor.

Example

$\frac{12}{20}$ is equivalent to $\frac{3}{5}$ since

$$\frac{12}{20} = \frac{4 \times 3}{4 \times 5} = \frac{3}{5}$$

Exercises

1. Express each of the following fractions in its simplest form:

a) $\frac{12}{16}$, b) $\frac{14}{21}$, c) $\frac{3}{6}$, d) $\frac{100}{45}$, e) $\frac{7}{9}$, f) $\frac{15}{55}$, g) $\frac{3}{24}$.

Answers

1. a) $\frac{3}{4}$, b) $\frac{2}{3}$, c) $\frac{1}{2}$, d) $\frac{20}{9}$, e) $\frac{7}{9}$, f) $\frac{3}{11}$, g) $\frac{1}{8}$.

2. Addition and subtraction of fractions

To add two fractions we first re-write each fraction so that they both have the same denominator. This denominator is chosen to be the **lowest common denominator**. The is the smallest number which is a multiple of both denominators. Then, the numerators only are added, and the result is divided by the lowest common denominator.

Example
Simplify a) $\frac{7}{16}+\frac{5}{16}$, b) $\frac{7}{16}+\frac{3}{8}$.

Solution
a) In this case the denominators of each fraction are already the same. The lowest common denominator is 16. We perform the addition by simply adding the numerators and dividing the result by the lowest common denominator. So, $\frac{7}{16}+\frac{5}{16}=\frac{7+5}{16}=\frac{12}{16}$. This answer can be expressed in the simpler form $\frac{3}{4}$ by cancelling the common factor 4.

b) To add these fractions we must rewrite them so that they have the same denominator. The lowest common denominator is 16 because this is the smallest number which is a multiple of both denominators. Note that $\frac{3}{8}$ is equivalent to $\frac{6}{16}$ and so we write $\frac{7}{16}+\frac{3}{8}=\frac{7}{16}+\frac{6}{16}=\frac{13}{16}$.

Example
Find $\frac{1}{2}+\frac{2}{3}+\frac{4}{5}$.

Solution
The smallest number which is a multiple of the given denominators is 30. We express each fraction with a denominator of 30.

$$\frac{1}{2}+\frac{2}{3}+\frac{4}{5}=\frac{15}{30}+\frac{20}{30}+\frac{24}{30}=\frac{59}{30}$$

Exercises
1. Evaluate each of the following:

a) $\frac{2}{3}+\frac{5}{4}$, b) $\frac{4}{9}-\frac{1}{2}$, c) $\frac{3}{4}+\frac{5}{6}$, d) $\frac{1}{4}+\frac{1}{3}+\frac{1}{2}$, e) $\frac{2}{5}-\frac{1}{3}-\frac{1}{10}$, f) $\frac{4}{5}+\frac{1}{3}-\frac{3}{4}$.

Answers
1. a) $\frac{23}{12}$, b) $-\frac{1}{18}$, c) $\frac{19}{12}$, d) $\frac{13}{12}$, e) $-\frac{1}{30}$, f) $\frac{23}{60}$.

3. Multiplication and division of fractions

Multiplication of fractions is more straightforward. We simply multiply the numerators to give a new numerator, and multiply the denominators to give a new denominator. For example

$$\frac{5}{7}\times\frac{3}{4}=\frac{5\times 3}{7\times 4}=\frac{15}{28}$$

Division is performed by inverting the second fraction and then multiplying. So,

$$\frac{5}{7}\div\frac{3}{4}=\frac{5}{7}\times\frac{4}{3}=\frac{20}{21}$$

Exercises
1. Find a) $\frac{4}{26}\times\frac{13}{7}$, b) $\frac{2}{11}\div\frac{3}{5}$, c) $\frac{2}{1}\times\frac{1}{2}$, d) $\frac{3}{7}\times\frac{2}{5}$, e) $\frac{3}{11}\times\frac{22}{5}$, f) $\frac{5}{6}\div\frac{4}{3}$.

Answers
1. a) $\frac{2}{7}$, b) $\frac{10}{33}$, c) 1, d) $\frac{6}{35}$, e) $\frac{6}{5}$, f) $\frac{5}{8}$.

Powers and roots

Introduction

Powers are used when we want to multiply a number by itself repeatedly.

1. Powers

When we wish to multiply a number by itself we use **powers**, or **indices** as they are also called.

For example, the quantity $7 \times 7 \times 7 \times 7$ is usually written as 7^4. The number 4 tells us the number of sevens to be multiplied together. In this example, the power, or index, is 4. The number 7 is called the **base**.

Example

$6^2 = 6 \times 6 = 36$. We say that '6 squared is 36', or '6 to the power 2 is 36'.

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. We say that '2 to the power 5 is 32'.

Your calculator will be pre-programmed to evaluate powers. Most calculators have a button marked x^y, or simply ˆ. Ensure that you are using your calculator correctly by verifying that $3^{11} = 177147$.

2. Square roots

When 5 is squared we obtain 25. That is $5^2 = 25$.

The reverse of this process is called **finding a square root**. The square root of 25 is 5. This is written as $\sqrt[2]{25} = 5$, or simply $\sqrt{25} = 5$.

Note also that when -5 is squared we again obtain 25, that is $(-5)^2 = 25$. This means that 25 has another square root, -5.

In general, a square root of a number is a number which when squared gives the original number. There are always two square roots of any positive number, one positive and one negative. However, negative numbers do not possess any square roots.

Most calculators have a square root button, probably marked $\surd$. Check that you can use your calculator correctly by verifying that $\sqrt{79} = 8.8882$, to four decimal places. Your calculator will only give the positive square root but you should be aware that the second, negative square root is -8.8882.

An important result is that the square root of a product of two numbers is equal to the product of the square roots of the two numbers. For example

$$\sqrt{16 \times 25} = \sqrt{16} \times \sqrt{25} = 4 \times 5 = 20$$

More generally,

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

However your attention is drawn to a common error which students make. It is not true that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$. Substitute some simple values for yourself to see that this cannot be right.

Exercises

1. Without using a calculator write down the value of $\sqrt{9 \times 36}$.
2. Find the square of the following: a) $\sqrt{2}$, b) $\sqrt{12}$.
3. Show that the square of $5\sqrt{2}$ is 50.

Answers

1. 18, (and also -18). 2. a) 2, b) 12.

3. Cube roots and higher roots

The cube root of a number, is the number which when cubed gives the original number. For example, because $4^3 = 64$ we know that the cube root of 64 is 4, written $\sqrt[3]{64} = 4$. All numbers, both positive and negative, possess a single cube root.

Higher roots are defined in a similar way: because $2^5 = 32$, the fifth root of 32 is 2, written $\sqrt[5]{32} = 2$.

Exercises

1. Without using a calculator find a) $\sqrt[3]{27}$, b) $\sqrt[3]{125}$.

Answers

1. a) 3, b) 5.

4. Surds

Expressions involving roots, for example $\sqrt{2}$ and $5\sqrt[3]{2}$ are also known as **surds**. Frequently, in engineering calculations it is quite acceptable to leave an answer in surd form rather than calculating its decimal approximation with a calculator.

It is often possible to write surds in equivalent forms. For example, $\sqrt{48}$ can be written as $\sqrt{3 \times 16}$, that is $\sqrt{3} \times \sqrt{16} = 4\sqrt{3}$.

Exercises

1. Write the following in their simplest surd form: a) $\sqrt{180}$, b) $\sqrt{63}$.
2. By multiplying numerator and denominator by $\sqrt{2} + 1$, show that

$$\frac{1}{\sqrt{2}-1} \quad \text{is equivalent to} \quad \sqrt{2}+1$$

Answers

1. a) $6\sqrt{5}$, b) $3\sqrt{7}$.

Scientific notation

Introduction

In engineering calculations numbers are often very small or very large, for example 0.00000345 and 870,000,000. To avoid writing lengthy strings of numbers a notation has been developed, known as **scientific notation** which enables us to write numbers much more concisely.

1. Scientific notation

In scientific notation each number is written in the form

$$a \times 10^n$$

where a is a number between 1 and 10 and n is a positive or negative whole number.

Some numbers in scientific notation are

$$5 \times 10^3, \qquad 2.67 \times 10^4, \qquad 7.90 \times 10^{-3}$$

To understand scientific notation you need to be aware that

$$10^1 = 10, \qquad 10^2 = 100, \qquad 10^3 = 1000, \qquad 10^4 = 10000, \qquad \text{and so on,}$$

and also that

$$10^{-1} = \frac{1}{10} = 0.1, \qquad 10^{-2} = \frac{1}{100} = 0.01, \qquad 10^{-3} = \frac{1}{1000} = 0.001, \qquad \text{and so on.}$$

You also need to remember how simple it is to multiply a number by powers of 10. For example to multiply 3.45 by 10, the decimal point is moved one place to the right to give 34.5. To multiply 29.65 by 100, the decimal point is moved two places to the right to give 2965. In general, to multiply a number by 10^n the decimal place is moved n places to the right if n is a positive whole number and n places to the left if n is a negative whole number. It may be necessary to insert additional zeros to make up the required number of digits.

Example
The following numbers are given in scientific notation. Write them out fully.

a) 5×10^3, b) 2.67×10^4, c) 7.90×10^{-3}.

Solution
a) $5 \times 10^3 = 5 \times 1000 = 5000$.

b) $2.67 \times 10^4 = 26700$.

c) $7.90 \times 10^{-3} = 0.00790$.

Example
Express each of the following numbers in scientific notation.

a) 5670000, b) 0.0098.

Solution
a) $5670000 = 5.67 \times 10^6$.

b) $0.0098 = 9.8 \times 10^{-3}$.

Exercises
1. Express each of the following in scientific notation.

a) 0.00254, b) 82, c) -0.342, d) 1000000.

Answers
1. a) 2.54×10^{-3}, b) 8.2×10, c) -3.42×10^{-1}, d) 1×10^6 or simply 10^6.

2. Using a calculator

Students often have difficulty using a calculator to deal with scientific notation. You may need to refer to your calculator manual to ensure that you are entering numbers correctly. You should also be aware that your calculator can display a number in lots of different forms including scientific notation. Usually a MODE button is used to select the appropriate format.

Commonly the EXP button is used to enter numbers in scientific notation. (EXP stands for exponent which is another name for a power). A number like 3.45×10^7 is entered as 3.45EXP 7 and might appear in the calculator window as $3.45\ ^{07}$. Alternatively your calculator may require you to enter the number as 3.45E7 and it may be displayed in the same way. You should seek help if in doubt.

Computer programming languages use similar notation. For example

$$8.25 \times 10^7 \qquad \text{may be programmed as} \qquad 8.25\text{E}7$$

and

$$9.1 \times 10^{-3} \qquad \text{may be programmed as} \qquad 9.1\text{E} - 3$$

Again, you need to take care and check the required syntax carefully.

A common error is to enter incorrectly numbers which are simply powers of 10. For example, the number 10^7 is erroneously entered as 10E7 which means 10×10^7, that is 10^8. The number 10^7, meaning 1×10^7, should be entered as $1E7$.

Check that you are using your calculator correctly by verifying that

$$(3 \times 10^7) \times (2.76 \times 10^{-4}) \times (10^5) = 8.28 \times 10^8$$

The laws of indices

Introduction

A **power**, or an **index**, is used to write a product of numbers very compactly. The plural of index is **indices**. In this leaflet we remind you of how this is done, and state a number of rules, or laws, which can be used to simplify expressions involving indices.

1. Powers, or indices

We write the expression

$$3 \times 3 \times 3 \times 3 \quad \text{as} \quad 3^4$$

We read this as 'three to the power four'.

Similarly

$$z \times z \times z = z^3$$

We read this as 'z to the power three' or 'z cubed'.

In the expression b^c, the **index** is c and the number b is called the **base**. Your calculator will probably have a button to evaluate powers of numbers. It may be marked x^y. Check this, and then use your calculator to verify that

$$7^4 = 2401 \qquad \text{and} \qquad 25^5 = 9765625$$

Exercises

1. Without using a calculator work out the value of

a) 4^2, b) 5^3, c) 2^5, d) $\left(\frac{1}{2}\right)^2$, e) $\left(\frac{1}{3}\right)^2$, f) $\left(\frac{2}{5}\right)^3$.

2. Write the following expressions more concisely by using an index.

a) $a \times a \times a \times a$, b) $(yz) \times (yz) \times (yz)$, c) $\left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right)$.

Answers

1. a) 16, b) 125, c) 32, d) $\frac{1}{4}$, e) $\frac{1}{9}$, f) $\frac{8}{125}$.

2. a) a^4, b) $(yz)^3$, c) $\left(\frac{a}{b}\right)^3$.

2. The laws of indices

To manipulate expressions involving indices we use rules known as the **laws of indices**. The laws should be used precisely as they are stated - do not be tempted to make up variations of your own! The three most important laws are given here:

First law

$$a^m \times a^n = a^{m+n}$$

When expressions with the same base are multiplied, the indices are added.

Example
We can write

$$7^6 \times 7^4 = 7^{6+4} = 7^{10}$$

You could verify this by evaluating both sides separately.

Example

$$z^4 \times z^3 = z^{4+3} = z^7$$

Second Law

$$\frac{a^m}{a^n} = a^{m-n}$$

When expressions with the same base are divided, the indices are subtracted.

Example
We can write

$$\frac{8^5}{8^3} = 8^{5-3} = 8^2 \qquad \text{and similarly} \qquad \frac{z^7}{z^4} = z^{7-4} = z^3$$

Third law

$$(a^m)^n = a^{mn}$$

Note that m and n have been multiplied to yield the new index mn.

Example

$$(6^4)^2 = 6^{4\times 2} = 6^8 \qquad \text{and} \qquad (e^x)^y = e^{xy}$$

It will also be useful to note the following important results:

$$a^0 = 1, \qquad a^1 = a$$

Exercises
1. In each case choose an appropriate law to simplify the expression:

a) $5^3 \times 5^{13}$, b) $8^{13} \div 8^5$, c) $x^6 \times x^5$, d) $(a^3)^4$, e) $\frac{y^7}{y^3}$, f) $\frac{x^8}{x^7}$.

2. Use one of the laws to simplify, if possible, $a^6 \times b^5$.

Answers
1. a) 5^{16}, b) 8^8, c) x^{11}, d) a^{12}, e) y^4, f) $x^1 = x$.

2. This cannot be simplified because the bases are not the same.

Engineering Maths First Aid Kit

2.2

Negative and fractional powers

Introduction

Sometimes it is useful to use negative and fractional powers. These are explained on this leaflet.

1. Negative powers

Sometimes you will meet a number raised to a negative power. This is interpreted as follows:

$$a^{-m} = \frac{1}{a^m}$$

This can be rearranged into the alternative form:

$$a^m = \frac{1}{a^{-m}}$$

Example

$$3^{-2} = \frac{1}{3^2}, \qquad \frac{1}{5^{-2}} = 5^2, \qquad x^{-1} = \frac{1}{x^1} = \frac{1}{x}, \qquad x^{-2} = \frac{1}{x^2}, \qquad 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$

Exercises

1. Write the following using only positive powers:

a) $\dfrac{1}{x^{-6}}$, b) x^{-12}, c) t^{-3}, d) $\frac{1}{4^{-3}}$, e) 5^{-17}.

2. Without using a calculator evaluate a) 2^{-3}, b) 3^{-2}, c) $\frac{1}{4^{-2}}$, d) $\frac{1}{2^{-5}}$, e) $\frac{1}{4^{-3}}$.

Answers

1. a) x^6, b) $\frac{1}{x^{12}}$, c) $\frac{1}{t^3}$, d) 4^3, e) $\frac{1}{5^{17}}$.

2. a) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$, b) $\frac{1}{9}$, c) 16, d) 32, e) 64.

2. Fractional powers

To understand fractional powers you first need to have an understanding of roots, and in particular square roots and cube roots. If necessary you should consult leaflet *1.2 Powers and Roots*.

When a number is raised to a fractional power this is interpreted as follows:

$$a^{1/n} = \sqrt[n]{a}$$

So,

$$\begin{array}{ll} a^{1/2} & \text{is a square root of } a \\ a^{1/3} & \text{is the cube root of } a \\ a^{1/4} & \text{is a fourth root of } a \end{array}$$

Example

$$3^{1/2} = \sqrt[2]{3}, \qquad 27^{1/3} = \sqrt[3]{27} \text{ or } 3, \qquad 32^{1/5} = \sqrt[5]{32} = 2,$$
$$64^{1/3} = \sqrt[3]{64} = 4, \qquad 81^{1/4} = \sqrt[4]{81} = 3$$

Fractional powers are useful when we need to calculate roots using a scientific calculator. For example to find $\sqrt[7]{38}$ we rewrite this as $38^{1/7}$ which can be evaluated using a scientific calculator. You may need to check your calculator manual to find the precise way of doing this, probably with the buttons x^y or $x^{1/y}$.

Check that you are using your calculator correctly by confirming that

$$38^{1/7} = 1.6814 \quad (4\,\text{dp})$$

More generally $a^{m/n}$ means $\sqrt[n]{a^m}$, or equivalently $(\sqrt[n]{a})^m$.

$$a^{m/n} = \sqrt[n]{a^m} \text{ or equivalently } \left(\sqrt[n]{a}\right)^m$$

Example

$$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4, \qquad \text{and} \qquad 32^{3/5} = (\sqrt[5]{32})^3 = 2^3 = 8$$

Exercises

1. Use a calculator to find a) $\sqrt[5]{96}$, b) $\sqrt[4]{32}$.
2. Without using a calculator, evaluate a) $4^{3/2}$, b) $27^{2/3}$.
3. Use the third law of indices to show that

$$a^{m/n} = \sqrt[n]{a^m}$$

and equivalently

$$a^{m/n} = \left(\sqrt[n]{a}\right)^m$$

Answers

1. a) 2.4915, b) 2.3784. 2. a) $4^{3/2} = 8$, b) $27^{2/3} = 9$.

Engineering Maths First Aid Kit

2.3

Removing brackets 1

Introduction

In order to simplify mathematical expressions it is frequently necessary to 'remove brackets'. This means to rewrite an expression which includes bracketed terms in an equivalent form, but without any brackets. This operation must be carried out according to certain rules which are described in this leaflet.

1. The associativity and commutativity of multiplication

Multiplication is said to be a **commutative** operation. This means, for example, that 4×5 has the same value as 5×4. Eitherway the result is 20. In symbols, xy is the same as yx, and so we can interchange the order as we wish.

Multiplication is also an **associative** operation. This means that when we want to multiply three numbers together such as $4 \times 3 \times 5$ it doesn't matter whether we evaluate 4×3 first and then multiply by 5, or evaluate 3×5 first and then multiply by 4. That is

$$(4 \times 3) \times 5 \qquad \text{is the same as} \qquad 4 \times (3 \times 5)$$

where we have used brackets to indicate which terms are multiplied first. Eitherway, the result is the same, 60. In symbols,we have

$$(x \times y) \times z \qquad \text{is the same as} \qquad x \times (y \times z)$$

and since the result is the same eitherway, the brackets make no difference at all and we can write simply $x \times y \times z$ or simply xyz. When mixing numbers and symbols we usually write the numbers first. So

$$\begin{aligned} 7 \times a \times 2 &= 7 \times 2 \times a \qquad \text{through commutativity} \\ &= 14a \end{aligned}$$

Example
Remove the brackets from a) $4(2x)$, b) $a(5b)$.

Solution
a) $4(2x)$ means $4 \times (2 \times x)$. Because of associativity of multiplication the brackets are unnecessary and we can write $4 \times 2 \times x$ which equals $8x$.

b) $a(5b)$ means $a \times (5b)$. Because of commutativity this is the same as $(5b) \times a$, that is $(5 \times b) \times a$. Because of associativity the brackets are unnecessary and we write simply $5 \times b \times a$ which equals $5ba$. Note that this is also equal to $5ab$ because of commutativity.

Exercises
1. Simplify

a) $9(3y)$, b) $(5x) \times (5y)$, c) $3(-2a)$, d) $-7(-9x)$, e) $12(3m)$, f) $5x(y)$.

Answers
1. a) $27y$, b) $25xy$, c) $-6a$, d) $63x$, e) $36m$, f) $5xy$.

2. Expressions of the form $a(b+c)$ and $a(b-c)$

Study the expression $4 \times (2+3)$. By working out the bracketed term first we obtain 4×5 which equals 20. Note that this is the same as multiplying both the 2 and 3 separately by 4, and then adding the results. That is

$$4 \times (2+3) = 4 \times 2 \quad + \quad 4 \times 3 = 8 + 12 = 20$$

Note the way in which the '4' multiplies both the bracketed numbers, '2' and '3'. We say that the '4' distributes itself over both the added terms in the brackets - *multiplication is distributive over addition*.

Now study the expression $6 \times (8-3)$. By working out the bracketed term first we obtain 6×5 which equals 30. Note that this is the same as multiplying both the 8 and the 3 by 6 before carrying out the subtraction:

$$6 \times (8-3) = 6 \times 8 \quad - \quad 6 \times 3 = 48 - 18 = 30$$

Note the way in which the '6' multiplies both the bracketed numbers. We say that the '6' distributes itself over both the terms in the brackets - *multiplication is distributive over subtraction*. Exactly the same property holds when we deal with symbols.

$$a(b+c) = ab + ac \qquad a(b-c) = ab - ac$$

Example
$4(5+x)$ is equivalent to $4 \times 5 \;+\; 4 \times x$ which equals $20 + 4x$.

$5(a-b)$ is equivalent to $5 \times a \;-\; 5 \times b$ which equals $5a - 5b$.

$7(x-2y)$ is equivalent to $7 \times x \;-\; 7 \times 2y$ which equals $7x - 14y$.

$-4(5+x)$ is equivalent to $-4 \times 5 \;+\; -4 \times x$ which equals $-20 - 4x$.

$-5(a-b)$ is equivalent to $-5 \times a \;-\; -5 \times b$ which equals $-5a + 5b$.

$-(a+b)$ is equivalent to $-a - b$.

Exercises
Remove the brackets from each of the following expressions simplifying your answers where appropriate.

1. $8(3+2y)$, 2. $7(-x+y)$, 3. $-7(-x+y)$, 4. $-(3+2x)$, 5. $-(3-2x)$,

6. $-(-3-2x)$, 7. $x(x+1)$, 8. $15(x+y)$, 9. $15(xy)$, 10. $11(m+3n)$.

Answers
1. $24+16y$, 2. $-7x+7y$, 3. $7x-7y$, 4. $-3-2x$, 5. $-3+2x$, 6. $3+2x$, 7. x^2+x,
8. $15x+15y$, 9. $15xy$ 10. $11m+33n$.

Removing brackets 2

Introduction

In this leaflet we show the correct procedure for writing expressions of the form $(a+b)(c+d)$ in an alternative form without brackets.

1. Expressions of the form $(a+b)(c+d)$

In the expression $(a+b)(c+d)$ it is intended that each term in the first bracket multiplies each term in the second.

$$(a+b)(c+d) = ac + bc + ad + bd$$

Example
Removing the brackets from $(5+a)(2+b)$ gives

$$5 \times 2 \;+\; a \times 2 \;+\; 5 \times b \;+\; a \times b$$

which simplifies to

$$10 + 2a + 5b + ab$$

.

Example
Removing the brackets from $(x+6)(x+2)$ gives

$$x \times x \;+\; 6 \times x \;+\; x \times 2 \;+\; 6 \times 2$$

which equals

$$x^2 + 6x + 2x + 12$$

which simplifies to

$$x^2 + 8x + 12$$

Example
Removing the brackets from $(x+7)(x-3)$ gives

$$x \times x \;+\; 7 \times x \;+\; x \times -3 \;+\; 7 \times -3$$

which equals

$$x^2 + 7x - 3x - 21$$

which simplifies to

$$x^2 + 4x - 21$$

Example
Removing the brackets from $(2x + 3)(x + 4)$ gives

$$2x \times x \ + \ 3 \times x \ + \ 2x \times 4 \ + \ 3 \times 4$$

which equals

$$2x^2 + 3x + 8x + 12$$

which simplifies to

$$2x^2 + 11x + 12$$

Occasionally you will need to square a bracketed expression. This can lead to errors. Study the following example.

Example
Remove the brackets from $(x + 1)^2$.

Solution
You need to be clear that when a quantity is squared it is multiplied by itself. So

$$(x + 1)^2 \qquad \text{means} \qquad (x + 1)(x + 1)$$

Then removing the brackets gives

$$x \times x \ + \ 1 \times x \ + \ x \times 1 \ + \ 1 \times 1$$

which equals

$$x^2 + x + x + 1$$

which simplifies to

$$x^2 + 2x + 1$$

Note that $(x + 1)^2$ is not equal to $x^2 + 1$, and more generally $(x + y)^2$ is not equal to $x^2 + y^2$.

Exercises
Remove the brackets from each of the following expressions simplifying your answers where appropriate.

1. a) $(x + 2)(x + 3)$, b) $(x - 4)(x + 1)$, c) $(x - 1)^2$, d) $(3x + 1)(2x - 4)$.
2. a) $(2x - 7)(x - 1)$, b) $(x + 5)(3x - 1)$, c) $(2x + 1)^2$, d) $(x - 3)^2$.

Answers

1. a) $x^2 + 5x + 6$, b) $x^2 - 3x - 4$, c) $x^2 - 2x + 1$, d) $6x^2 - 10x - 4$.
2. a) $2x^2 - 9x + 7$, b) $3x^2 + 14x - 5$, c) $4x^2 + 4x + 1$, d) $x^2 - 6x + 9$.

Factorising simple expressions

Introduction

Before studying this material you must be familiar with the process of 'removing brackets' as outlined on leaflets 2.3 & 2.4. This is because factorising can be thought of as reversing the process of removing brackets. When we factorise an expression it is written as a product of two or more terms, and these will normally involve brackets.

1. Products and Factors

To obtain the **product** of two numbers they are multiplied together. For example the product of 3 and 4 is 3×4 which equals 12. The numbers which are multiplied together are called factors. We say that 3 and 4 are both factors of 12.

Example
The product of x and y is xy.

The product of $5x$ and $3y$ is $15xy$.

Example
$2x$ and $5y$ are factors of $10xy$ since when we multiply $2x$ by $5y$ we obtain $10xy$.

$(x+1)$ and $(x+2)$ are factors of x^2+3x+2 because when we multiply $(x+1)$ by $(x+2)$ we obtain x^2+3x+2.

3 and $x-5$ are factors of $3x-15$ because

$$3(x-5) = 3x-15$$

2. Common Factors

Sometimes, if we study two expressions to find their factors, we might note that some of the factors are the same. These factors are called **common factors**.

Example
Consider the numbers 18 and 12.

Both 6 and 3 are factors of 18 because $6 \times 3 = 18$.

Both 6 and 2 are factors of 12 because $6 \times 2 = 12$.

So, 18 and 12 share a common factor, namely 6.

In fact 18 and 12 share other common factors. Can you find them ?

Example
The number 10 and the expression $15x$ share a common factor of 5.

Note that $10 = 5 \times 2$, and $15x = 5 \times 3x$. Hence 5 is a common factor.

Example
$3a^2$ and $5a$ share a common factor of a since

$3a^2 = 3a \times a$ and $5a = 5 \times a$. Hence a is a common factor.

Example
$8x^2$ and $12x$ share a common factor of $4x$ since

$8x^2 = 4x \times 2x$ and $12x = 3 \times 4x$. Hence $4x$ is a common factor.

3. Factorising

To factorise an expression containing two or more terms it is necessary to look for factors which are common to the different terms. Once found, these common factors are written outside a bracketed term. It is ALWAYS possible to check your answers when you factorise by simply removing the brackets again, so you shouldn't get them wrong.

Example
Factorise $15x + 10$.

Solution
First we look for any factors which are common to both $15x$ and 10. The common factor here is 5. So the original expression can be written

$$15x + 10 = 5(3x) + 5(2)$$

which shows clearly the common factor. This common factor is written outside a bracketed term, the remaining quantities being placed inside the bracket:

$$15x + 10 = 5(3x + 2)$$

and the expression has been factorised. We say that the factors of $15x + 10$ are 5 and $3x + 2$. Your answer can be checked by showing

$$5(3x + 2) = 5(3x) + 5(2) = 15x + 10$$

Exercises
Factorise each of the following:

1. $10x + 5y$, 2. $21 + 7x$, 3. $xy - 8x$, 4. $4x - 8xy$

Answers
1. $5(2x + y)$, 2. $7(3 + x)$, 3. $x(y - 8)$, 4. $4x(1 - 2y)$.

Simplifying fractions

Introduction

Fractions involving symbols occur very frequently in engineering mathematics. It is necessary to be able to simplify these and rewrite them in different but equivalent forms. On this leaflet we revise how these processes are carried out. It will be helpful if you have already seen leaflet *1.1 Fractions*.

1. Expressing a fraction in its simplest form

An algebraic fraction can always be expressed in different, yet **equivalent** forms. A fraction is expressed in its **simplest form** by cancelling any factors which are common to both the numerator and the denominator. You need to remember that factors are multiplied together.

For example, the two fractions

$$\frac{7a}{ab} \qquad \text{and} \qquad \frac{7}{b}$$

are equivalent. Note that there is a common factor of a in the numerator and the denominator of $\frac{7a}{ab}$ which can be cancelled to give $\frac{7}{b}$.

> To express a fraction in its simplest form, any factors which are common to both the numerator and the denominator are cancelled

Notice that cancelling is equivalent to dividing the top and the bottom by the common factor.

It is also important to note that $\frac{7}{b}$ can be converted back to the equivalent fraction $\frac{7a}{ab}$ by multiplying both the numerator and denominator of $\frac{7}{b}$ by a.

> A fraction is expressed in an equivalent form by multiplying both top and bottom by the same quantity, or dividing top and bottom by the same quantity

Example
The two fractions

$$\frac{10y^2}{15y^5} \qquad \text{and} \qquad \frac{2}{3y^3}$$

are equivalent. Note that

$$\frac{10y^2}{15y^5} = \frac{2 \times 5 \times y \times y}{3 \times 5 \times y \times y \times y \times y \times y}$$

and so there are common factors of 5 and $y \times y$. These can be cancelled to leave $\dfrac{2}{3y^3}$.

Example
The fractions

$$\frac{(x-1)(x+3)}{(x+3)(x+5)} \qquad \text{and} \qquad \frac{(x-1)}{(x+5)}$$

are equivalent. In the first fraction, the common factor $(x+3)$ can be cancelled.

Example
The fractions

$$\frac{2a(3a-b)}{7a(a+b)} \qquad \text{and} \qquad \frac{2(3a-b)}{7(a+b)}$$

are equivalent. In the first fraction, the common factor a can be cancelled. Nothing else can be cancelled.

Example
In the fraction

$$\frac{a-b}{a+b}$$

there are no common factors which can be cancelled. Neither a nor b is a factor of the numerator. Neither a nor b is a factor of the denominator.

Example
Express $\dfrac{5x}{2x+1}$ as an equivalent fraction with denominator $(2x+1)(x-7)$.

Solution
To achieve the required denominator we must multiply both top and bottom by $(x-7)$. That is

$$\frac{5x}{2x+1} = \frac{(5x)(x-7)}{(2x+1)(x-7)}$$

If we wished, the brackets could now be removed to write the fraction as $\dfrac{5x^2-35x}{2x^2-13x-7}$.

Exercises
1. Express each of the following fractions in its simplest form:

a) $\frac{12xy}{16x}$, b) $\frac{14ab}{21a^2b^2}$, c) $\frac{3x^2y}{6x}$, d) $\frac{3(x+1)}{(x+1)^2}$, e) $\frac{(x+3)(x+1)}{(x+2)(x+3)}$, f) $\frac{100x}{45}$, g) $\frac{a+b}{ab}$.

Answers
1. a) $\frac{3y}{4}$, b) $\frac{2}{3ab}$, c) $\frac{xy}{2}$, d) $\frac{3}{x+1}$, e) $\frac{x+1}{x+2}$, f) $\frac{20x}{9}$, g) cannot be simplified. Whilst both a and b are factors of the denominator, neither a nor b is a factor of the numerator.

Addition and subtraction

Introduction

Fractions involving symbols occur very frequently in engineering mathematics. It is necessary to be able to add and subtract them. On this leaflet we revise how these processes are carried out. An understanding of writing fractions in equivalent forms is necessary. (See leaflet *2.7 Simplifying fractions*.)

1. Addition and subtraction of fractions

To add two fractions we must first re-write each fraction so that they both have the same denominator. The denominator is called the **lowest common denominator**. It is the simplest expression which is a multiple of both of the original denominators. Then, the numerators only are added, and the result is divided by the lowest common denominator.

Example
Express as a single fraction

$$\frac{7}{a} + \frac{9}{b}$$

Solution
Both fractions must be written with the same denominator. To achieve this, note that if the numerator and denominator of the first are both multiplied by b we obtain $\frac{7b}{ab}$. This is equivalent to the original fraction - it is merely written in a different form. If the numerator and denominator of the second are both multiplied by a we obtain $\frac{9a}{ab}$. Then the problem becomes

$$\frac{7b}{ab} + \frac{9a}{ab}$$

In this form, both fractions have the same denominator. The lowest common denominator is ab.

Finally we add the numerators and divide the result by the lowest common denominator:

$$\frac{7b}{ab} + \frac{9a}{ab} = \frac{7b + 9a}{ab}$$

Example
Express as a single fraction

$$\frac{2}{x+3} + \frac{5}{x-1}$$

Solution
Both fractions can be written with the same denominator if both the numerator and denominator of the first are multiplied by $x - 1$ and if both the numerator and denominator of the second are multiplied by $x + 3$. This gives

$$\frac{2}{x+3} + \frac{5}{x-1} = \frac{2(x-1)}{(x+3)(x-1)} + \frac{5(x+3)}{(x+3)(x-1)}$$

Then, adding the numerators gives

$$\frac{2(x-1)+5(x+3)}{(x+3)(x-1)}$$

which, by simplifying the numerator, gives

$$\frac{7x+13}{(x+3)(x-1)}$$

Example
Find $\dfrac{3}{x+1} + \dfrac{2}{(x+1)^2}$

Solution
The simplest expression which is a multiple of the original denominators is $(x+1)^2$. This is the lowest common denominator. Both fractions must be written with this denominator.

$$\frac{3}{x+1} + \frac{2}{(x+1)^2} = \frac{3(x+1)}{(x+1)^2} + \frac{2}{(x+1)^2}$$

Adding the numerators and simplifying we find

$$\frac{3(x+1)}{(x+1)^2} + \frac{2}{(x+1)^2} = \frac{3x+3+2}{(x+1)^2} = \frac{3x+5}{(x+1)^2}$$

Exercises
1. Express each of the following as a single fraction:

a) $\frac{3}{4} + \frac{1}{x}$, b) $\frac{1}{a} - \frac{2}{5b}$, c) $\frac{2}{x^2} + \frac{1}{x}$, d) $2 + \frac{1}{3x}$.

2. Express as a single fraction:

a) $\frac{2}{x+1} + \frac{3}{x+2}$, b) $\frac{2}{x+3} + \frac{5}{(x+3)^2}$, c) $\frac{3x}{x-1} + \frac{1}{x}$, d) $\frac{1}{x-5} - \frac{3}{x+2}$, e) $\frac{1}{2x+1} - \frac{7}{x+3}$.

Answers
1. a) $\frac{3x+4}{4x}$, b) $\frac{5b-2a}{5ab}$, c) $\frac{2+x}{x^2}$, d) $\frac{6x+1}{3x}$.

2. a) $\frac{5x+7}{(x+1)(x+2)}$, b) $\frac{2x+11}{(x+3)^2}$, c) $\frac{3x^2+x-1}{x(x-1)}$, d) $\frac{17-2x}{(x+2)(x-5)}$, e) $-\frac{13x+4}{(x+3)(2x+1)}$.

Multiplication and division

Introduction

Fractions involving symbols occur very frequently in engineering mathematics. It is necessary to be able to multiply and divide them. On this leaflet we revise how these processes are carried out. It will be helpful if you have already seen leaflet *1.1 Fractions*.

1. Multiplication and division of fractions

Multiplication of fractions is straightforward. We simply multiply the numerators to give a new numerator, and multiply the denominators to give a new denominator.

Example
Find

$$\frac{4}{7} \times \frac{a}{b}$$

Solution
Simply multiply the two numerators together, and multiply the two denominators together.

$$\frac{4}{7} \times \frac{a}{b} = \frac{4a}{7b}$$

Example
Find

$$\frac{3ab}{5} \times \frac{7}{6a}$$

Solution

$$\frac{3ab}{5} \times \frac{7}{6a} = \frac{21ab}{30a}$$

which, by cancelling common factors, can be simplified to $\frac{7b}{10}$.

Division is performed by inverting the second fraction and then multiplying.

Example
Find $\frac{3}{2x} \div \frac{6}{5y}$.

Solution

$$\begin{aligned}\frac{3}{2x} \div \frac{6}{5y} &= \frac{3}{2x} \times \frac{5y}{6} \\ &= \frac{15y}{12x} \\ &= \frac{5y}{4x}\end{aligned}$$

Example
Find $\dfrac{3}{x+1} \div \dfrac{x}{(x+1)^2}$.

Solution

$$\begin{aligned}\frac{3}{x+1} \div \frac{x}{(x+1)^2} &= \frac{3}{x+1} \times \frac{(x+1)^2}{x} \\ &= \frac{3(x+1)^2}{x(x+1)} \\ &= \frac{3(x+1)}{x}\end{aligned}$$

Exercises
1. Find a) $\frac{1}{3} \times \frac{x}{2}$, b) $\frac{2}{x+1} \times \frac{x}{x-3}$, c) $-\frac{1}{4} \times \frac{3}{5}$, d) $\left(-\frac{1}{x}\right) \times \left(\frac{2}{5y}\right)$, e) $\frac{x+1}{2(x+3)} \times \frac{8}{x+1}$.
2. Simplify

$$\frac{3}{x+2} \div \frac{x}{2x+4}$$

3. Simplify

$$\frac{x+2}{(x+5)(x+4)} \times \frac{x+5}{x+2}$$

4. Simplify

$$\frac{3}{x} \times \frac{3}{y} \times \frac{1}{z}$$

5. Find $\dfrac{4}{3} \div \dfrac{16}{x}$.

Answers
1. a) $\frac{x}{6}$, b) $\frac{2x}{(x+1)(x-3)}$, c) $-\frac{3}{20}$, d) $-\frac{2}{5xy}$, e) $\frac{4}{x+3}$.
2. $\frac{6}{x}$, 3. $\frac{1}{x+4}$, 4. $\frac{9}{xyz}$, 5. $\frac{x}{12}$.

Rearranging formulas 1

Introduction

The ability to rearrange formulas or rewrite them in different ways is an important skill in engineering. This leaflet will explain how to rearrange some simple formulas. Leaflet 2.11 deals with more complicated examples.

1. The subject of a formula

Most engineering students will be familiar with Ohm's law which states that $V = IR$. Here, V is a voltage drop, R is a resistance and I is a current. If the values of R and I are known then the formula $V = IR$ enables us to calculate the value of V. In the form $V = IR$, we say that the **subject** of the formula is V. Usually the subject of a formula is on its own on the left-hand side. You may also be familiar with Ohm's law written in either of the forms

$$I = \frac{V}{R} \qquad \text{and} \qquad R = \frac{V}{I}$$

In the first case I is the subject of the formula whilst in the second case R is the subject. If we know values of V and R we can use $I = \frac{V}{R}$ to find I. On the other hand, if we know values of V and I we can use $R = \frac{V}{I}$ to find R. So you see, it is important to be able to write formulas in different ways, so that we can make a particular variable the subject.

2. Rules for rearranging, or transposing, a formula

You can think of a formula as a pair of balanced scales. The quantity on the left is equal to the quantity on the right. If we add an amount to one side of the scale pans, say the left one, then to keep balance we must add the same amount to the pan on the right. Similarly if we take away an amount from the left, we must take the same amount away from the pan on the right. The same applies to formulas. If we add an amount to one side, we must add the same to the other to keep the formula valid. If we subtract an amount from one side we must subtract the same amount from the other. Furthermore, if we multiply the left by any amount, we must multiply the right by the same amount. If we divide the left by any amount we must divide the right by the same amount. When you are trying to rearrange, or **transpose**, a formula, keep these operations clearly in mind.

To transpose or rearrange a formula you may

- add or subtract the same quantity to or from both sides
- multiply or divide both sides by the same quantity

Later, we shall see that a further group of operations is allowed, but first get some practice with these Examples and Exercises.

Example
Rearrange the formula $y = x + 8$ in order to make x the subject instead of y.

Solution
To make x the subject we must remove the 8 from the right. So, we subtract 8 from the right, but we remember that we must do the same to the left. So

$$\begin{aligned} \text{if} \qquad y &= x + 8, \qquad \text{subtracting 8 yields} \\ y - 8 &= x + 8 - 8 \\ y - 8 &= x \end{aligned}$$

We have x on its own, although it is on the right. This is no problem since if $y - 8$ equals x, then x equals $y - 8$, that $x = y - 8$. We have succeeded in making x the subject of the formula.

Example
Rearrange the formula $y = 3x$ to make x the subject.

Solution
The reason why x does not appear on its own is that it is multiplied by 3. If we divide $3x$ by 3 we obtain $\frac{3x}{3} = x$. So, we can obtain x on its own by dividing both sides of the formula by 3.

$$\begin{aligned} y &= 3x \\ \frac{y}{3} &= \frac{3x}{3} \\ &= x \end{aligned}$$

Finally $x = \frac{y}{3}$ and we have succeeded in making x the subject of the formula.

Example
Rearrange $y = 11 + 7x$ to make x the subject.

Solution
Starting from $y = 11 + 7x$ we subtract 11 from each side to give $y - 11 = 7x$. Then, dividing both sides by 7 gives $\frac{y-11}{7} = x$. Finally $x = \frac{y-11}{7}$.

Exercises
1. Transpose each of the following formulas to make x the subject.

a) $y = x - 7$, b) $y = 2x - 7$, c) $y = 2x + 7$, d) $y = 7 - 2x$, e) $y = \frac{x}{5}$.

2. Transpose each of the following formulas to make v the subject.

a) $w = 3v$, b) $w = \frac{1}{3}v$, c) $w = \frac{v}{3}$, d) $w = \frac{2v}{3}$, e) $w = \frac{2}{3}v$.

Answers
1. a) $x = y + 7$, b) $x = \frac{y+7}{2}$, c) $x = \frac{y-7}{2}$, d) $x = \frac{7-y}{2}$, e) $x = 5y$.
2. a) $v = \frac{w}{3}$, b) $v = 3w$, c) same as b), d) $v = \frac{3w}{2}$, e) same as d).

Rearranging formulas 2

Introduction

This leaflet develops the work started on leaflet 2.11, and shows how more complicated formulas can be rearranged.

1. Further transposition

Remember that when you are trying to rearrange, or **transpose**, a formula, the following operations are allowed.

- add or subtract the same quantity to or from both sides
- multiply or divide both sides by the same quantity

A further group of operations is also permissible.

A formula remains balanced if we perform the same operation to both sides of it. For example, we can square both sides, we can square-root both sides. We can find the logarithm of both sides. Study the following examples.

Example
Transpose the formula $p = \sqrt{q}$ to make q the subject.

Solution
Here we need to obtain q on its own. To do this we must find a way of removing the square root sign. This can be achieved by squaring both sides since

$$(\sqrt{q})^2 = q$$

So,

$$\begin{aligned} p &= \sqrt{q} \\ p^2 &= q \qquad \text{by squaring both sides} \end{aligned}$$

Finally, $q = p^2$, and we have succeeded in making q the subject of the formula.

Example
Transpose $p = \sqrt{a+b}$ to make b the subject.

Solution

$$\begin{aligned} p &= \sqrt{a+b} \\ p^2 &= a+b \qquad \text{by squaring both sides} \\ p^2 - a &= b \end{aligned}$$

Finally, $b = p^2 - a$, and we have succeeded in making b the subject of the formula.

Example
Make x the subject of the formula $v = \frac{k}{\sqrt{x}}$.

Solution

$$\begin{aligned} v &= \frac{k}{\sqrt{x}} \\ v^2 &= \frac{k^2}{x} \qquad \text{by squaring both sides} \\ xv^2 &= k^2 \qquad \text{by multiplying both sides by } x \\ x &= \frac{k^2}{v^2} \qquad \text{by dividing both sides by } v^2 \end{aligned}$$

and we have succeeded in making x the subject of the formula.

Example
Transpose the formula $T = 2\pi\sqrt{\frac{\ell}{g}}$ for ℓ.

Solution
This must be carried out carefully, in stages, until we obtain ℓ on its own.

$$\begin{aligned} T &= 2\pi\sqrt{\frac{\ell}{g}} \\ \frac{T}{2\pi} &= \sqrt{\frac{\ell}{g}} \qquad \text{by dividing both sides by } 2\pi \\ \left(\frac{T}{2\pi}\right)^2 &= \frac{\ell}{g} \qquad \text{by squaring both sides} \\ \ell &= g\left(\frac{T}{2\pi}\right)^2 \end{aligned}$$

Exercises
1. Make r the subject of the formula $V = \frac{4}{3}\pi r^3$.

2. Make x the subject of the formula $y = 4 - x^2$.

3. Make s the subject of the formula $v^2 = u^2 + 2as$

Answers
1. $r = \sqrt[3]{\frac{3V}{4\pi}}$. 2. $x = \sqrt{4-y}$. 3. $s = \frac{v^2-u^2}{2a}$.

Solving linear equations

Introduction

Equations occur in all branches of engineering. They always involve one or more unknown quantities which we try to find when we **solve** the equation. The simplest equations to deal with are **linear equations**. On this leaflet we describe how these are solved.

1. A linear equation

Linear equations are those which can be written in the form

$$ax + b = 0$$

where x is the unknown value, and a and b are known numbers. The following are all examples of linear equations.

$$3x + 2 = 0, \qquad -5x + 11 = 0, \qquad 3x - 11 = 0$$

The unknown does not have to have the symbol x, other letters can be used.

$$3t - 2 = 0, \qquad 7z + 11 = 0, \qquad 3w = 0$$

are all linear equations.

Sometimes you will come across a linear equation which at first sight might not appear to have the form $ax + b = 0$. The following are all linear equations. If you have some experience of solving linear equations, or of transposing formulas, you will be able to check that they can all be written in the standard form.

$$\frac{x-7}{2} + 11 = 0, \qquad \frac{2}{x} = 8, \qquad 6x - 2 = 9$$

2. Solving a linear equation

To solve a linear equation it will be helpful if you know already how to transpose or rearrange formulas. (See leaflets *2.10 & 2.11 Rearranging formulas*, for information about this if necessary).

When solving a linear equation we try to make the unknown quantity the subject of the equation. To do this we may

- add or subtract the same quantity to or from both sides
- multiply or divide both sides by the same quantity

Example
Solve the equation $x + 7 = 18$.

Solution
We try to obtain x on its own on the left hand side.

$$\begin{aligned} x + 7 &= 18 \\ x &= 18 - 7 \qquad \text{by subtracting 7 from both sides} \\ x &= 11 \end{aligned}$$

We have solved the equation and found the solution: $x = 11$. The solution is that value of x which can be substituted into the original equation to make both sides the same. You can, and should, check this. Substituting $x = 11$ in the left-hand side of the equation $x + 7 = 18$ we find $11 + 7$ which equals 18, the same as the right-hand side.

Example
Solve the equation $5x + 11 = 22$.

Solution

$$\begin{aligned} 5x + 11 &= 22 \\ 5x &= 22 - 11 \qquad \text{by subtracting 11 from both sides} \\ x &= \frac{11}{5} \qquad \text{by dividing both sides by 5} \end{aligned}$$

Example
Solve the equation $13x - 2 = 11x + 17$.

Solution

$$\begin{aligned} 13x - 2 &= 11x + 17 \\ 13x - 11x - 2 &= 17 \qquad \text{by subtracting } 11x \text{ from both sides} \\ 2x - 2 &= 17 \\ 2x &= 17 + 2 \qquad \text{by adding 2 to both sides} \\ 2x &= 19 \\ x &= \frac{19}{2} \end{aligned}$$

Exercises
1. Solve the following linear equations.

a) $4x + 8 = 0$, b) $3x - 11 = 2$, c) $8(x + 3) = 64$, d) $7(x - 5) = -56$, e) $3c - 5 = 14c - 27$.

Answers
1. a) $x = -2$, b) $x = \frac{13}{3}$, c) $x = 5$, d) $x = -3$, e) $c = 2$.