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Modelling the Sectoral Allocation of Labour in Open Economy Models

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Abstract

Indivisible labour is not the only type of nonconvexity affecting labour supply decisions. Another type of nonconvexity arises in economies with sectors whenever individuals can work in only one sector at a time. I introduce this restriction into an open economy model with a tradeable and a nontradeable sector, and I use lotteries to convexify the consumption possibilities set. This approach implies that the aggregate elasticity of labour supply becomes infinite. I compare the performance of the model with an analogous model in which the labour supply elasticity is finite. I find that the infinite labour supply elasticity helps explain the

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persistence of net exports. However, all the other consequences of the labour supply elasticity for the model-implied second-order moments depend on whether the pricing assumption is Producer Currency Pricing (PCP) or Local Currency Pricing (LCP).

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1 Introduction

This paper studies the performance of a two-country model with tradeable and nontradeable sectors in which individuals cannot supply their labour services to both sectors at the same time. Accounting for the non-convexity arising from this restriction is important for two reasons. First, in real life most people do not or cannot hold two jobs at the same time. Secondly, macroeconomists have developed models with non-convexities which reconcile low individual labour supply elasticities with the observed large fluctuations of aggregate hours over the business cycle. In my model, the aggregate labour supply elasticity is infinite, as in a classic indivisible labour model. I find that the labour supply elasticity influences the response of wages and prices to exogenous shocks, but ultimately its impact on the model's performance depend on the pricing assumption.

This paper contributes to the literature by examining the implications of a non-standard assumption regarding the allocation of hours worked between sectors. Many open economy models have two sectors, one producing internationally traded goods and one producing nontradeable goods, so they must also specify how individuals choose to allocate their labour time between the two sectors. The standard assumption is that only the sum of hours worked enters the utility function. As a result, the representative agent is completely indifferent between, say, working 20 hours a week in a tradeable sector firm plus 20 hours in a nontradeable sector firm, and working 40 hours a week in only one of the two firms. Instead I consider an economy in which individual choices are restricted, either work in one sector or the other, so the consumption possibilities set is non-convex. This environment was first introduced by Rogerson (1988b). Like him, I assume employment lotteries with complete markets and I show that the utility function features both the intensive (hours) and the extensive (participation rates) margins of labour supply. These preferences imply that all the adjustment in the labour supply occurs through the extensive, not the intensive, margin, and the Frisch elasticity of the labour supply is infinite.

As it is well known, the observed large fluctuations in aggregate hours imply that the aggregate labour supply elasticity must be large (Prescott 2005). Moreover, a large labour supply elasticity is important for monetary shocks to have persistent effects on output (Chari, Kehoe and McGrattan 2000). However, estimated elasticities from microeconometric studies are well below the calibrated values in macroeconomic models. Seminal work by Hansen (1985) and Rogerson (1988a) showed that these opposing facts can be reconciled by assuming that individual agents are only allowed to make the choice as to whether to be employed or not, but cannot adjust the number of hours worked. In this environment, the elasticity of the aggregate labour supply is infinite. Critics of Rogerson's aggregation consider it to be at odds with microeconomic observations, because it relies on employment lotteries with complete markets. However, recently Ljungqvist and Sargent (2005, 2011), and Rogerson and Wallenius (2009) have explored an alternative 'time-averaging' aggregation, according to which individuals face a $\{0, 1\}$ employment choice in each period and choose what fraction of their lifetime to work. So far, this debate has not influenced the open economy literature, despite the fact that a special kind of labour indivisibility arises quite naturally in economies with sectors (Rogerson 1988b).

Nevertheless, several contributions have uncovered a number of open economy results which depend on the labour supply elasticity. Basu and Kollmann (2013) and Kollmann (2010) show that a high labour supply elasticity is necessary to ensure that the real exchange rate depreciates after an increase in government expenditure, consistently with the empirical evidence. Using a two-sector dependent economy model, Morshed and Turnovsky (2011) show that the elasticity of labour supply affects the speed of convergence of the real exchange rate to its long-run equilibrium value. Corsetti, Martin and Pesenti (2007) analyse the reallocation of endogenous product varieties to the most productive country following a shock. They find that this reallocation can be considerable, but only if the labour supply elasticity is so high that relative wages are not affected by the shock. In contrast to these contributions, I do not focus on a specific effect or statistics, but instead I investigate the impact of the labour supply elasticity on the second-order moments of several variables. I consider both demand (money and government expenditure) and supply-type (productivity) shocks, and I show that the consequences of varying the labour supply elasticity are dependent on the pricing assumption.

As in Obstfeld and Rogoff (1995), my model features monopolistic competition and price rigidity. An important issue in this literature is the choice of currency of invoicing. This choice is important because in a two-country, two-currency world it is possible to model price rigidity in different ways. One way, for example, is to assume that the law of one price holds and that prices are sticky in the currency of the producer (producer currency pricing or PCP). This assumption is made, among others, by Obstfeld and Rogoff (1995, 2000, 2007), Corsetti and Pesenti (2001), Galí and Monacelli (2005), and Benigno (2009). Another possibility is to assume that prices are sticky in the currency of the destination market (local currency pricing or LCP). This assumption is made, for example, by Betts and Devereux (1996, 2000), Kollmann (2001), Chari, Kehoe and McGrattan (2002), Benigno and Thoenissen (2003), and Sutherland (2005). To date, the choice of pricing assumption and the degree of exchange rate pass-through into import prices are still open questions in the literature. I follow the approach of Corsetti and Pesenti (2005) and I allow the pass-through elasticity to be either one or zero. This enables me to consider both PCP and LCP as special cases of a single specification.

Since the Frisch elasticity of labour supply cannot be calibrated freely, I compare the performance of the infinite elasticity model with an analogous model in which individuals supply their labour services to both sectors at the same time, and the labour supply elasticity is finite. I find that the infinite labour supply elasticity dampens the response of wages and prices to exogenous shocks. The higher is the labour supply elasticity, the smaller is price adjustment, and the higher is the persistence of the series. However, the consequences of varying the labour supply elasticity for the model-implied second-order moments depend on whether the pricing assumption is PCP or LCP. The two variables that are most affected by the labour supply elasticity are net exports and the terms of trade. The infinite Frisch elasticity increases the volatility of the terms of trade in the PCP scenario, but decreases it in the LCP scenario. Moreover, a finite and relatively low labour supply elasticity is important to generate countercyclical net exports as in the data, but this only happens in the LCP case. The only consequence of the infinite labour supply elasticity that is not dependent on the pricing assumption is the improved persistence of net exports.

The remainder of the paper is as follows. Section 2 illustrates the model, and Section 3 the alternative assumption that individuals supply labour contemporaneously to both sectors. The calibration of the model is described in Section 4. Sections 5 and 6 explain the findings, and Section 7 concludes.

2 The model

The model includes features such as Calvo-style price rigidity, nontradeable goods and home bias in consumption. The elasticity of exchange rate pass-through is a free parameter of the model, which nests both PCP and LCP as special cases.

The world economy consists of two countries, Home and Foreign. Both countries have two sectors, and in each sector there exists a continuum of monopolistic firms, each of them producing a single differentiated product, or brand. The notation is as follows. The firms and the goods they produce are indexed by $f_{TH} \in [0,1]$ for the Home tradeable sector and $f_N \in [0,1]$ for the Home nontradeable sector. In the Foreign country, they are indexed by $f^*_{TF} \in [0,1]$ and $f^*_N \in [0,1]$ respectively. All Foreign variables and indexes are denoted with stars. Prices of individual varieties are denoted with lower cases, aggregate prices with upper cases. Steady state variables have a zero time index.

Firms

Each firm has a fixed probability of changing its prices at date t. All prices are set in the currency of the buyer, thus tradeable goods firms in both countries set two different prices, one for the Home market and one for the Foreign market, denominated in the respective local currencies. However, the degree of exchange rate pass-through is not necessarily zero, since export prices can adjust to changes in the nominal exchange rate.

More formally, I follow the approach of Corsetti and Pesenti (2005), and assume that the local currency prices of exports of Home and Foreign tradeable varieties f_{TH} and f^*_{TF} are given, respectively, by:

$$p^*_{TH,t}\left(f_{TH}\right) = \frac{\widetilde{p}_{TH,t}\left(f_{TH}\right)}{e_t^{\zeta}} \ , \qquad p_{TF,t}\left(f^*_{TF}\right) = e_t^{\zeta}\widetilde{p}^*_{TF,t}\left(f^*_{TF}\right) \ , \qquad (1)$$

where e is the nominal exchange rate (price of the Home currency in terms of the Foreign currency), ζ is the pass-through elasticity, constant by assumption, and $\widetilde{p}_{TH}\left(f_{TH}\right)$ and $\widetilde{p}^*_{TF}\left(f^*_{TF}\right)$ are predetermined components that are not adjusted to variations in the exchange rate during period t. Thus, if ζ is equal to one the exchange rate pass-through is complete, and if ζ is equal to zero the pass-through is zero.

For example, a Home tradeable sector firm f_{TH} chooses the price $p_{TH,t}(f_{TH})$ of domestic sales, and the predetermined component $\widetilde{p}_{TH,t}(f_{TH})$ of the export price, by maximising the present discounted value of profits:

$$E_t \sum_{j=0}^{\infty} (\varphi\beta)^j Q_{t,t+j} \left[\begin{array}{c} \frac{p_{TH,t}(f_{TH})}{P_{t+j}} \cdot y_{TH,t+j|t}(f_{TH}) \\ +e_{t+j} \frac{p^*_{TH,t+j}(f_{TH})}{P_{t+j}} y^*_{TH,t+j|t}(f_{TH}) \\ -\frac{W_{TH,t+j}}{P_{t+j}} \cdot \widetilde{h}_{TH,t+j|t}(f_{TH}) \end{array} \right], \tag{2}$$

subject to:

$$\begin{aligned} y_{TH,t+j|t}(f_{TH}) &= \left(\frac{p_{TH,t}(f_{TH})}{P_{TH,t+j}}\right)^{-\eta} C_{TH,t+j}, \\ y^*_{TH,t+j|t}(f_{TH}) &= \left(\frac{p^*_{TH,t+j|t}(f_{TH})}{P^*_{TH,t+j}}\right)^{-\eta} C^*_{TH,t+j}, \\ p^*_{TH,t+j|t}(f_{TH}) &= \widetilde{p}_{TH,t}(f_{TH})\, e^{-\zeta}_{t+j}, \end{aligned} \tag{3}$$

where $Q_{t,t+j} = \frac{u'(C_{t+j})}{u'(C_t)}$, and $(\varphi)^j$ is the probability that $p_{TH,t}(f_{TH})$ and $\widetilde{p}_{TH,t}(f_{TH})$ still apply at the future date $t+j$. The variables $y_{TH,t+j|t}(f_{TH})$ and $y^*_{TH,t+j|t}(f_{TH})$ denote the Home and Foreign demands for good f_{TH}, and $\widetilde{h}_{TH,t+j|t}(f_{TH})$ denotes the total labour input used by the firm, if the prices decided at t still apply at date $t+j$.

Output sold at Home and abroad is produced using a common plant or production function:

$$y_{TH,t}\left(f_{TH}\right)+y_{TH,t}^{*}\left(f_{TH}\right)=z_{TH,t}\cdot\widetilde{h}_{TH,t}\left(f_{TH}\right)^{\alpha}\ , \tag{4}$$

where the parameter α allows for decreasing returns to labour, and z_{TH} represents technology.

In the Foreign country, the production function and maximization problem of the tradeable sector firms f_{TF}^{*} are the same as in the Home country. All parameters are assumed to be the same in both countries and sectors. The pricing behaviour and production functions of nontradeable sector firms f_N and f_N^{*} are as described in this section, except for the fact that nontradeable firms serve only their own domestic market and do not engage in price discrimination.

Consumption indexes

Preferences over tradeable and nontradeable goods in the Home country are specified as follows:[1]

$$C_t=\left[(1-\gamma)^{\frac{1}{\phi}}\left(C_{T,t}\right)^{\frac{\phi-1}{\phi}}+\gamma^{\frac{1}{\phi}}\left(C_{N,t}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}\ . \tag{5}$$

The Home aggregator for tradeable goods consumption is:

$$C_{T,t}=\left[(1-\delta)^{\frac{1}{\theta}}\left(C_{TH,t}\right)^{\frac{\theta-1}{\theta}}+\delta^{\frac{1}{\theta}}\left(C_{TF,t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}\ . \tag{6}$$

[1] Preferences in the Foreign country are described by the same aggregators.

The consumption sub-indices for the individual varieties are CES aggregators, with constant elasticity of substitution η. Price indexes are defined as the minimal expenditures needed to buy one unit of the corresponding consumption aggregators.

Government budget constraint and money supply

The Home and Foreign governments purchase only nontradeable goods produced in their own country. The budget constraint of the Home government at date t is given by:[2]

$$M_t - M_{t-1} = P_{N,t}G_t + TR_t \ , \tag{7}$$

where G is a CES aggregator of varieties f_N, with the same elasticity of substitution η.

Individual preferences and labour supply

The Home and Foreign countries are populated by a continuum of homogeneous individuals uniformly distributed on $[0,1]$. I discuss only the Home maximisation problem, since it is the same in both countries. In each period the individual chooses consumption, real money balances $\frac{M}{P}$ and hours worked in each sector. Let $\mathbf{h}_{TH}$ and $\mathbf{h}_N$ denote total hours supplied to all firms in sectors TH and N. Total time available to an employed individual is normalized to one, and total time available to an unemployed individual

[2] The Foreign government budget constraint and the public expenditure aggregator are entirely analogous.

is denoted with τ. An individual who works incurs a fixed participation or commuting cost ψ. Because of the restriction that labour cannot be supplied in both sectors simultaneously, the individual's consumption possibilities set $\boldsymbol{X}$ in any given period is nonconvex:

$$\boldsymbol{X} = \left\{ \left(C,\, \frac{M}{P},\, \mathbf{h}_{TH},\, \mathbf{h}_N \right) : C \geq 0,\, \frac{M}{P} \geq 0,\, 0 \leq \mathbf{h}_{TH} \leq 1 - \psi,\, 0 \leq \mathbf{h}_N \leq 1 - \psi,\, \mathbf{h}_{TH} \cdot \mathbf{h}_N = 0 \right\} .$$

The individual's utility function[3] is:

$$U_0 = E_0 \sum\nolimits_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} + \frac{\chi}{1-\varepsilon} \left(\frac{M_t}{P_t} \right)^{1-\varepsilon} + \upsilon \left(\mathbf{h}_{TH,t},\, \mathbf{h}_{N,t} \right) \right] , \qquad (8)$$

where:

$$\upsilon \left(\mathbf{h}_{TH,t},\, \mathbf{h}_{N,t} \right) = \begin{cases} \frac{\kappa}{\omega} \left(1 - \psi - \mathbf{h}_{TH,t} \right)^{\omega} & \text{if } \mathbf{h}_{TH,t} \neq 0 \ , \\ \frac{\kappa}{\omega} \left(1 - \psi - \mathbf{h}_{N,t} \right)^{\omega} & \text{if } \mathbf{h}_{N,t} \neq 0 \ , \\ \frac{\kappa}{\omega} \left(\tau \right)^{\omega} & \text{if } \mathbf{h}_{TH,t} = \mathbf{h}_{N,t} = 0 \ . \end{cases}$$

The consumption set can be convexified by adding lotteries over the choice of working in the two sectors, and with complete markets the decentralized

[3] I choose these functional forms because Rogerson's (1988b) aggregation requires separable preferences, and because analogous functional forms (but not the nonconvexity) can be found in the literature; for example, Obstfeld and Rogoff (1995) or Benigno and Thoenissen (2003).

equilibrium reproduces the socially optimal allocation.[4] In this environment the individual maximises her expected utility, which is given by:

$$U_0 = E_0 \sum\nolimits_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \frac{C_t^{1-\sigma}-1}{1-\sigma} + \frac{\chi}{1-\varepsilon} \left(\frac{M_t}{P_t}\right)^{1-\varepsilon} + n_{TH,t} \cdot \frac{\kappa}{\omega} \left(1 - \psi - \mathbf{h}_{TH,t}\right)^{\omega} \\ + n_{N,t} \cdot \frac{\kappa}{\omega} \left(1 - \psi - \mathbf{h}_{N,t}\right)^{\omega} \\ + \left(1 - n_{TH,t} - n_{N,t}\right) \cdot \frac{\kappa}{\omega} \left(\tau\right)^{\omega} \end{array} \right], \tag{9}$$

where n_{TH} and n_N are the probabilities of working in the tradeable and nontradeable sectors. Because of the law of large numbers, these are equal to the fractions of individuals at the aggregate level.

The aggregation based on employment lotteries has attracted some objections (Ljungqvist and Sargent 2011), but on the other hand the utility function (9) possesses several advantages. First, it disentangles both margins of labour supply, hours and participation rates. Second, since the probabilities enter linearly, it can be interpreted as average or expected utility. Third, this specification does not impose that sectors pay the same wage.

In order to examine the implications for the labour supply elasticity, it is necessary to specify the budget constraint. Individuals trade in a one-period non-contingent real bond, denominated in units of the Home tradeable goods consumption index, sold at the price P_T. Similarly to Benigno (2001), individuals must pay a small cost in order to undertake a position in the

[4] Thus the allocations are the same as those chosen by a utilitarian household with a unit mass of homogeneous individuals. The household assigns a fraction of its members to sector TH and another fraction to sector N, pools its members' labour incomes and ensures that each one receives the same level of consumption.

international asset market.[5] This cost is assumed to be a payment in exchange for intermediation services, offered by financial firms located in both the Home and the Foreign country. Individuals pay this cost only to firms located in their own country.

The period-t budget constraint of the individual in the Home country is as follows:

$$\begin{aligned} B_t P_{T,t} + \frac{\nu}{C_0} B_t^2 P_{T,t} + M_t &\leq (1+r_{t-1}) B_{t-1} P_{T,t} + M_{t-1} \\ &+ TR_t - P_t C_t + n_{TH,t} W_{TH,t} \mathbf{h}_{TH,t} + n_{N,t} W_{N,t} \mathbf{h}_{N,t} \\ &+ \int_0^1 \Pi_{TH,t}(f_{TH})\, df_{TH} + \int_0^1 \Pi_{N,t}(f_N)\, df_N + R_t \ , \end{aligned} \tag{10}$$

where B is the internationally traded bond, $\frac{\nu}{C_0} B$ is the cost of holding one unit of the bond, which depends on the positive parameter ν, r is the real interest rate, TR are government transfers, W_{TH} and W_N are the wages paid in the tradeable and nontradeable sector respectively, $\Pi_{TH}(f_{TH})$ and $\Pi_N(f_N)$ are the profits that the individual receives from firms f_{TH} (tradeable sector) and f_N (nontradeable sector), and R represents the rents generated by the financial intermediaries. The internationally traded bond B is in zero net supply worldwide. Wages are flexible.

When both participation rates and hours of work are choice variables the assumption that preferences are separable has important consequences. By

[5] This assumption ensures stationarity of the model and a well-defined steady state, as demonstrated by Schmitt-Grohe and Uribe (2003).

combining a few first order conditions of the maximization problem we obtain:

$$\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{TH,t}\right)^{\omega}+\kappa\left(1-\psi-\mathbf{h}_{TH,t}\right)^{\omega-1}\mathbf{h}_{TH,t}=\frac{\kappa}{\omega}\left(\tau\right)^{\omega}\ , \tag{11}$$

$$\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{N,t}\right)^{\omega}+\kappa\left(1-\psi-\mathbf{h}_{N,t}\right)^{\omega-1}\mathbf{h}_{N,t}=\frac{\kappa}{\omega}\left(\tau\right)^{\omega}\ . \tag{12}$$

Equations (11) and (12) above must have a unique solution, but the solution must be the same in the steady state and in each date t. Therefore, in this model hours worked in the two sectors are always constant and equal to each other.[6] This result in turn implies that the first order conditions with respect to the labour effort reduce to only one equation:

$$\kappa\left(1-\psi-\mathbf{h}_{0}\right)^{\omega-1}C_t^{\sigma}=\frac{W_{TH,t}}{P_t}=\frac{W_{N,t}}{P_t}\ , \tag{13}$$

where $\mathbf{h}_0$ is endogenously constant. Notice that in Hansen's (1985) model $\mathbf{h}_0$ is exogenously given instead. Wages are equalized between sectors, and in this model output demand determines the amount of the labour input. The aggregate labour supply,[7] i.e. the supply of $n_t \equiv n_{TH,t} + n_{N,t}$ holding wealth constant, is infinitely elastic, as is the supply of $n_{TH,t}$ and $n_{N,t}$.

[6] It is possible to ensure that hours worked in the two sectors are different by specifying a different participation cost ψ in the two sectors.

[7] In Appendix A.2 I investigate whether the infinite elasticity is due to the employment lottery or the homogeneity of individuals. I show that heterogeneity *per se* does not guarantee a finite elasticity of labour supply, and what matters in a model with this type of non-convexity is how the aggregate variables are derived from the preferences of heterogeneous individuals. In a social planner solution it is possible to have a finite labour supply elasticity if individuals are heterogeneous. In a competitive equilibrium with employment lotteries the elasticity of labour supply is infinite, even with agent heterogeneity.

3 If labour is supplied in both sectors simultaneously

The standard assumption in the literature is that individuals can work contemporaneously in both the tradeable and nontradeable sectors. For comparability purposes I keep the same functional forms in both scenarios. The utility function and budget constraint are as follows:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} + \frac{\chi}{1-\varepsilon} \left(\frac{M_t}{P_t} \right)^{1-\varepsilon} + \frac{\kappa}{\omega} \left(1 - \psi - \mathbf{h}_{TH,t} - \mathbf{h}_{N,t}\right)^{\omega} \right] , \tag{14}$$

$$\begin{aligned} B_t P_{T,t} + \frac{\nu}{C_0} B_t^2 P_{T,t} + M_t \leq{}& \left(1 + r_{t-1}\right) B_{t-1} P_{T,t} + M_{t-1} \\ & + TR_t - P_t C_t + W_t \left(\mathbf{h}_{TH,t} + \mathbf{h}_{N,t}\right) \\ + \int_0^1 \Pi_{TH,t} \left(f_{TH}\right) df_{TH} &+ \int_0^1 \Pi_{N,t} \left(f_N\right) df_N + R_t \ . \end{aligned} \tag{15}$$

Since hours worked enter additively, the individual is indifferent between working in one sector or both, provided the aggregate labour supply $\mathbf{h}_t \equiv \mathbf{h}_{TH,t} + \mathbf{h}_{N,t}$ is the same. Notice that in an interior solution sectors must pay the same wage.

It may be possible to interpret (14) as the utility function of a stand-in household, whose hours of work equal aggregate hours in the economy. There are however some unresolved issues with this interpretation. The utility function (14) does not distinguish between the intensive and the extensive margins of labour supply, however, if $\mathbf{h}_{TH,t}$ and $\mathbf{h}_{N,t}$ are to be interpreted as aggregate hours, they must be the outcome of choices made on both margins. If, for example, we regard the hours in (14) as the product of participation rates times hours worked per person, then this specification is neither the average nor the expected utility of the members of the household. More generally, it is not possible to see how the intensive and extensive margins determine the aggregate hours in (14) without a formal derivation of the utility of the stand-in household from individual preferences.

To examine the implications of (14) for the labour supply elasticity, consider the first order condition with respect to the labour effort:

$$\kappa\left(1-\psi-\mathbf{h}_t\right)^{\omega-1} C_t^{\sigma}=\frac{W_t}{P_t}\ . \tag{16}$$

The Frisch elasticity of the aggregate labour supply is $\frac{1}{1-\omega}\frac{1-\psi-\mathbf{h}_t}{\mathbf{h}_t}$. Given $\mathbf{h}_0$, the choice of ω determines its steady state value. Therefore, the labour supply (for a given level of wealth) is upward sloping.[8] Firms decide how aggregate hours worked are allocated between the two sectors.

4 Parameterization

The parameterization of the model is shown in Table 1.

[8]Once the Frisch elasticity is chosen, the actual values of κ and ψ are irrelevant to the dynamics of the log-linearized model. Notice that if $\omega=1$ the elasticity of labour supply becomes infinite.

TABLE 1 HERE

The parameter σ is the same as in Chari, Kehoe and McGrattan (2002). Given σ, I choose ϵ so that the consumption elasticity of money demand is equal to one, and I choose κ and ψ so that hours worked in the steady state are equal to 324.8/1369.[9]

The elasticity of substitution between tradeable and nontradeable goods is as in Obstfeld and Rogoff (2005). I choose a value for the elasticity of substitution between domestic and foreign tradeables that is somehow in the middle of the range of values in the literature. The preference weight for nontradeables γ is set between the values suggested by Obstfeld and Rogoff (2007) and Benigno and Thoenissen (2003), and the parametrization of δ, the preference weight for Foreign-produced tradeables, is as in Obstfeld and Rogoff (2004). I calibrate the steady state ratios of exogenous technology so that the ratio of Home to Foreign tradeable output is equal to one, and the Home and Foreign ratios of tradeable to nontradeable output[10] are equal to 0.2.

The intermediation cost parameter ν is chosen so that the spread in the nominal interest rates approximates the value suggested by Benigno (2009). The parameter η implies that the steady state markup is about 1.15, and the probabilities of not changing prices imply an average price duration of about

[9] These numbers are average hours worked in a year and total hours available, taken from Burnside and Eichenbaum (1996).

[10] The ratio of value added in manufacturing over the value added in services is approximately equal to 0.2 in the US. Source: own calculations based on the Groningen 60-Industry Database, http://www.ggdc.net.

one year. The elasticity of output with respect to hours is calibrated so that, given the mark-up, in the steady state the share of wages in output is equal to 0.7.

The growth rates of technology, the money growth rates and government expenditures are all assumed to be exogenously given by AR(1) processes, with zero unconditional means (except for the technology processes). The calibrated parameters of the exogenous processes are taken from Chari, Kehoe and McGrattan (2002) and Corsetti, Dedola and Leduc (2008), and are the same for both countries. Chari, Kehoe and McGrattan (2002) calibrate the variance of the monetary shocks so that their model reproduces the standard deviation of US GDP. This method gives a different calibrated value in each specification of the model. Since I want to keep the volatility of the money shocks constant in all specifications, I proceed as follows. I compute the standard deviation of the monetary shocks so as to match the standard deviation of US GDP (given all the other parameters in Table 1), under four different scenarios: finite and infinite elasticity, LCP and PCP. I then set the standard deviation of the monetary shocks equal to the average of these four values.

I solve the model numerically using Uhlig's "Toolkit" algorithm (1999). The numerical solution is obtained by log-linearising the equations around a deterministic equilibrium or steady state. I assume that in the steady state bond holdings are zero.

5 Results

I illustrate the performance of the model of Section (2) against the data and against the standard assumption that individuals supply labour contemporaneously to both sectors, in which case I assume that preferences are as in Section (3) and so the labour supply elasticity is finite. I consider two alternative values for the Frisch elasticity[11], 1.5 and 0.75, and I report second-order moments of the finite and the infinite elasticity models in Tables 2 and 3. I consider both pricing assumptions, LCP and PCP.

TABLES 2 AND 3 HERE

An important issue to consider beforehand is the measurement of the aggregate labour input. In the model of Section (2), all variation in the labour input is due to variation in the extensive margin, or changes in participation rates, so I measure the aggregate labour input with n_t. On the other hand, if individuals supply labour contemporaneously to both sectors and preferences are as in Section (3), all variation in the labour input is due variation in the intensive margin, or changes in hours, so the aggregate labour input is $\mathbf{h}_t$. I choose to measure the aggregate labour input in the data with aggregate hours, which are the product of average weekly hours and employment, and

[11] These are steady state values. I choose these two values because most estimates in the macro literature lie in this range. Raffo (2008) reports that the range of estimates for the Frisch elasticity of labour supply is between 1 and 1.5 at the aggregate level. Based on their survey of the literature, Chetty et al. (2011) recommend calibrating macro models to match a Frisch elasticity of aggregate hours of 0.75.

On the other hand, some authors in the literature assume that the disutility from labour is linear, so the Frisch labour supply elasticity is infinite (for example, Cooke 2010).

therefore reflect changes along both margins.[12]

The other variables of interest are real aggregate output, which is defined as $Y_t \equiv P_{TH,0}Y_{TH,t} + P^*_{TH,0}Y^*_{TH,t} + P_{N,0}Y_{N,t}$, while total tradeable output is $Y^{Tot}_{TH,t} \equiv Y_{TH,t} + Y^*_{TH,t} = C_{TH,t} + C^*_{TH,t}$. The real exchange rate is the ratio of Foreign to Home aggregate price indexes $RER_t \equiv (e_t P^*_t)/P_t$, and the (Home) terms of trade is the relative price of imports over exports:

$$T_t \equiv \frac{P_{TF,t}}{e_t P^*_{TH,t}} \tag{17}$$

Finally, net exports are measured as the ratio of real net exports to real GDP, $NX_t \equiv \left(P^*_{TH,0}Y^*_{TH,t} - P_{TF,0}Y_{TF,t}\right)/Y_t$.

As it is possible to see from Tables 2 and 3, the finite and the infinite elasticity models do not generate the same statistics, therefore the labour supply elasticity affects the performance of open economy models, particularly along some dimensions. Net exports are one of the variables most affected by this elasticity. A high Frisch elasticity causes the volatility of net exports to increase, but this happens only under PCP. Under LCP, net exports are countercyclical if the Frisch elasticity is low, but the effect of the elasticity on the co-movement between net exports and output disappears under PCP. Countercyclical net exports are an important feature of the data, and the literature has found that the ability to reproduce a negative correlation between net exports and output crucially depends on the utility function (Raffo

[12] This choice is consistent with many studies, including the indivisible labour literature. For example, Hansen (1985) considers total hours (i.e. aggregate) for persons at work in non-agricultural industries. However, other studies measure $\mathbf{h}_t$ with employment data (for example, Chari, Kehoe and McGrattan 2002).

2008, Janko 2011). In particular, preferences à la Greenwood, Hercowitz and Huffman (1988) are considered superior in this respect. However, Table 2 suggests that the pricing assumption is another important consideration, and that other preferences, such as separable utility, can also generate countercyclical net exports under LCP.

The other variable most affected by the labour supply elasticity is the terms of trade. A high Frisch elasticity increases the volatility of the terms of trade in the PCP scenario, but decreases it in the LCP scenario. Moreover, under PCP a high Frisch elasticity improves the persistence of the terms of trade, but there is no improvement under LCP. Overall, the consequences of the labour supply elasticity for the model-implied second-order moments depend on whether the pricing assumption is PCP or LCP. The only consequence of the infinite labour supply elasticity that is robust to the pricing assumption is the improved persistence of net exports.

On the other hand, there are some facts that are common to both the finite and the infinite elasticity models. Under both PCP and LCP, the standard deviations of consumption, output, employment and the nominal exchange rate are fairly close to the data. The cross-correlations of consumption and hours with output are also fairly close to the data. However, both the finite and the infinite elasticity models do not match the data along several dimensions. They do not generate enough volatility in the real exchange rate and generate too much volatility in the terms of trade. The standard deviation of net exports is well above or well below the data, in the PCP and LCP scenarios, respectively. They generate cross-correlations of the terms of trade, real and nominal exchange rate that are well away from the data. Finally, the model-generated series are not as persistent as the data.

Given the focus of this paper on the tradeable and nontradeable sectors, I also report sectoral statistics in Tables 2 and 3. In the data, the tradeable sector is represented by manufacturing, and the nontradeable sector by the service sector.[13] Under both LCP and PCP, the sectoral statistics generated by the infinite elasticity model are similar to the ones obtained with a finite Frisch elasticity. However, only in the PCP scenario the infinite and the finite elasticity models are able to generate more volatile employment and output in the tradeable sector than in the nontradeable sector.[14]

Naturally, we can ask why the finite and the infinite elasticity models do not generate exactly the same statistics. What consequences can be expected by varying the Frisch elasticity in open economy models? I will answer this question in the paragraphs that follow. I will explain first why the slope of the labour supply matters for the transmission of shocks, and in the next Section I will focus on the individual variables.

[13] The sectoral statistics presented in Tables 2 and 3 differ from the ones in Devereux and Hnatkovska (2012). This is because they report the properties of sectoral shares, while I compute the statistics using sectoral output levels.

[14] Notice that, since some manufacturing output is nontradeable, and some services are actually traded internationally, the data is an imprecise measure of the theoretical tradeable and nontradeable output levels. To some extent, this is true for all sectoral classifications of the data. Therefore, it is more sensible to investigate the ability of the model to reproduce the same qualitative pattern as in the data (higher volatilities in the tradeable sector), rather than its ability to replicate the data moments quantitatively. I discuss this measurement error in Povoledo (2013).

The Frisch elasticity is the elasticity of the labour supply curve, holding wealth constant. Therefore, the larger is this elasticity and the more pronounced is the response of employment after a shock. This fact is confirmed by Tables 2 and 3: both sectoral and aggregate employment are more volatile when the Frisch elasticity is higher. But notice that the larger is the Frisch elasticity and the flatter is the labour supply curve, so not only the response of employment is magnified, but also the response of wages is reduced. Since wages affect marginal costs, the lower is the response of wages, the lower is the response of prices after a shock, because firms optimally choose not to raise their prices much if wages do not rise much. Therefore, the higher is the Frisch elasticity, the less responsive are prices.

Since the Frisch elasticity controls the responses of prices after a shock, it fundamentally affects the response of output, at the sectoral as well as the aggregate level. To understand how output is affected by the Frisch elasticity, it is essential to distinguish between supply-type and demand-type shocks.[15] After a positive demand-type shock, such as a positive monetary or government expenditure shock, labour demand increases, putting upward pressure on wages and prices. But the smallest is the increase in prices, the bigger is the effect of the demand shock on output. Therefore, a comparatively high Frisch elasticity amplifies the effect of demand-type shocks on output. On the other hand, after a positive supply-type shock, such as a positive technology shock, labour demand falls, putting downward pressure on wages and

[15] In explaining how output is affected by the Frisch elasticity, for simplicity I only consider shocks originating in the same country and sector.

therefore prices. The strongest is the fall in prices, the bigger is the effect of the supply-type shock on output. Therefore, a relatively high Frisch elasticity reduces the effect of supply-type shocks on output. In conclusion, the impact of the Frisch elasticity on output volatility depends on which shocks are the main source of business cycle fluctuations. Tables 2 and 3 show that the infinite Frisch elasticity causes output to become more volatile: this fact suggests that in the model demand-type shocks are the main cause of business cycles. This intuition is confirmed by a formal variance decomposition exercise that I present in Section 6.

The Frisch elasticity of labour supply also affects the persistence of the model-generated series. Except for the persistence of the shocks, the only other mechanism ensuring persistence is the Calvo price stickiness. If prices were fully flexible the adjustment towards the steady state would be very rapid. As explained above, if the Frisch elasticity of labour supply is relatively high, wages, and therefore marginal costs, do not change much after a shock. As a result, the firms that are allowed to change their price after a shock will optimally choose a small adjustment, and ultimately a small price adjustment gives persistence. Tables 2 and 3 confirm this explanation.[16]

Moreover, since the Frisch elasticity of labour supply affects the persistence, it can also affect the cross correlations between variables. For example, consider any two variables which move together in the same direction, after any shock and at all horizons. If the Frisch elasticity is relatively high, as

[16] The only exception is the autocorrelation of the terms of trade, which actually goes down if the Frisch elasticity increases.

explained above the adjustment towards the steady state is slower, so the two variables in this example will stay positively correlated at longer horizons. As a result, their correlation coefficient will increase. Of course, not all variables move in the same direction at all horizons and after all shocks. This example merely serves to illustrate why the Frisch elasticity matters for some cross correlation coefficients, as shown by Tables 2 and 3, but its impact on any given coefficient cannot be generalised, instead, it must be investigated on a case-by-case basis.

6 Discussion

To further understand the results of Tables 2 and 3 it is important to ascertain which shocks are the main sources of business cycle fluctuations, and how the macroeconomic variables respond to them. The former task can be achieved by performing a variance decomposition exercise, and the latter by inspecting the impulse responses.

TABLE 4 HERE

The variance decompositions of the model of Section (2) are shown in Table 4. For most variables, Home and Foreign money shocks are the main cause of fluctuations, but the impact of technology and government expenditure shocks on aggregate and sectoral employment and output is significant. Since nontradeables make up the largest component of aggregate output, a large proportion of the variance of aggregate output is explained by government expenditure on nontradeable goods. However, the sum of Home and Foreign money shocks always explains the largest share of the variance of most variables, even of those that are significantly affected by technology and government expenditure shocks. Therefore, for the sake of concision, I only present the responses to Home money shocks under both PCP and LCP.[17]

FIGURE 1 HERE

Figure 1 shows the responses of consumption, the terms of trade and the real and nominal exchange rates. A positive Home monetary shock causes a nominal depreciation of the Home currency, which is more pronounced in the LCP scenario. Because of price rigidity, the nominal depreciation is accompanied by a real depreciation. Under PCP, the exchange rate pass-through into import prices is full, so the currency depreciation causes an increase in Home import prices plus a fall in export prices, and as a result the terms of trade increases. On the other hand, under LCP there is no exchange rate pass-through, thus the nominal depreciation causes the terms of trade to fall.[18]

As noted in Section 5, the finite and the infinite elasticity models have very different implications for the volatility of the terms of trade. I will now provide an explanation of this fact, focusing on monetary shocks only as these explain almost 90% of the variance of the terms of trade. Consider LCP first. After a positive Home monetary shock, the nominal depreciation raises the

[17] The responses to Foreign shocks are symmetric because the parameterization is the same for the Home and Foreign economies. The responses to all the other shocks are available on request.

[18] See Obstfeld and Rogoff (2000) for an analysis of the implications of the PCP and LCP assumptions for the terms of trade.

denominator of Equation 17. Home prices also increase, so the denominator of Equation 17 increases also because of the increase in the predetermined component of export prices. As explained in Section 5, the response of prices depends on the Frisch elasticity. The lower is the Frisch elasticity, the larger is the increase in the predetermined component of export prices, so the more pronounced is the fall in the terms of trade immediately after the positive Home monetary shock. Therefore, under LCP the terms of trade is more volatile if the Frisch elasticity is relatively low. This fact is confirmed by Table 2.

Next, consider a positive Home monetary shock under PCP. In this case, a nominal depreciation does not affect the denominator of Equation 17, instead it raises the numerator proportionally.[19] But because the predetermined component of export prices always affects the denominator, the rise in Home prices now dampens the terms of trade increase, so a lower Frisch elasticity lessens the responsiveness of the terms of trade to monetary shocks. As a result, under PCP the terms of trade is less volatile if the Frisch elasticity is relatively low, which is confirmed by Table 3.

FIGURES 2 AND 3 HERE

The responses of aggregate employment, aggregate output and net exports, which are shown in Figure 2, are also affected by the pricing assumption. Under PCP, Home employment, output and net exports all benefit from expenditure-switching (the shift of foreign and domestic demand towards

[19] Obstfeld and Rogoff (2000), p. 120.

Home tradeable goods). Because of expenditure-switching, the higher is the volatility of the terms of trade, and the higher is the volatility of net exports. On the other hand, under LCP nominal exchange rate movements are not passed onto international prices, so there is no expenditure-switching.[20] As a result, after a positive Home monetary shock, output and employment increase considerably less and net exports become negative instead. Therefore, the absence of expenditure-switching is crucial for net exports to be countercyclical, as in the data. However, notice that, at longer horizons, the responses of net exports and output have the same sign. So the slower is the adjustment towards the steady state, the less negative is the correlation. Hence, in order to ensure that the correlation between net exports and output stays negative under LCP, we could select a comparatively low Frisch elasticity (see Table 2) because it helps to speed up the adjustment towards the steady state.

7 Conclusion

The challenge of building macroeconomic models that are consistent with the microeconometric evidence has generated renewed interest on indivisible labour. However, indivisible labour is only one type of nonconvexity affecting the labour supply. In models with sectors, such as many international macro models, a nonconvexity arises whenever individuals cannot work in two or more sectors at the same time.

[20] The impact of expenditure-switching under PCP can be deduced from Figure 3. The immediate increase in tradeable sector output and labour input is almost three times as large in the PCP case than in the LCP case.

It is fair to say that open economy macroeconomics has not been affected yet by the debate on the microfoundations of aggregate labour supply. Perhaps the explanation is that a model with two countries and two sectors is inherently larger than the closed economy, one sector models typically analysed in the existing literature on nonconvexities in labour supply. Analytical tractability is understandably a deciding factor.

This paper shows that it is possible to deal with the restriction that individuals cannot contemporaneously work in two sectors at the same time without sacrificing analytical tractability. To simplify aggregation I use lotteries with complete markets. One drawback of this approach is that the elasticity of labour supply becomes infinite. However, I show that the inability to calibrate this elasticity to any finite value of choice does not compromise the performance of the model, since what matters more in models of this type is ultimately the choice of pricing assumption. The only effect of the infinite elasticity that is robust to the pricing assumption is the increase in the persistence of net exports, although it is not enough to match the data.

One advantage of this approach is that the utility function features both the intensive (hours) and the extensive (participation rates) margins of labour supply. Since individuals cannot work in two sectors at the same time, exogenous shocks trigger a reallocation of workers between sectors, which may be a costly or lengthy process. Therefore, it may be interesting to extend the model by considering such costs or delays, and to analyse how the transmission of shocks or the Balassa-Samuelson effect are affected by them. I leave these issues for future research.

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Table 1: Parameter values

Utility	$\beta = 0.99$, $\sigma = \varepsilon = 5$, $\mathbf{h}_0 = 0.24$
Consumption indexes	$\phi = 1$, $\theta = 1$, $\gamma = 0.70$, $\delta = 0.30$
Asset market	$\nu = 0.005$
Firms	$\varphi = 0.75$, $\eta = 7.88$, $\alpha = 0.8$ $\zeta = 0$ (LCP) or $\zeta = 1$ (PCP)
Exogenous processes: $\widehat{x}_{j,t} = \overline{x}_j + \rho_j \cdot \widehat{x}_{j,t-1} + \epsilon_j$	
Money growth	$\rho = 0.68$, $\mathrm{var}(\epsilon) = \mathrm{var}(\epsilon^*) = (0.0151)^2$, $\mathrm{corr}(\epsilon, \epsilon^*) = 0.50$
Tradeable technology	$\rho = 0.95$, $\mathrm{var}(\epsilon) = \mathrm{var}(\epsilon^*) = (0.007)^2$, $\mathrm{corr}(\epsilon, \epsilon^*) = 0.25$
Nontradeable technology	$\rho = 0.95$, $\mathrm{var}(\epsilon) = \mathrm{var}(\epsilon^*) = (0.007)^2$, $\mathrm{corr}(\epsilon, \epsilon^*) = 0.25$
Government expenditure:	$\rho = 0.97$, $\mathrm{var}(\epsilon) = \mathrm{var}(\epsilon^*) = (0.01)^2$, $\mathrm{corr}(\epsilon, \epsilon^*) = 0$

Table 2: Business cycle statistics under LCP

Standard deviations	C	Y	n	NX	T	RER	e	
US data	0.97	1.26	1.78	0.36	2.23	6.12	6.01	
Infinite elasticity	1.08	1.17	1.64	0.07	3.78	4.55	5.86	
Frisch = 1.5	1.05	1.11	1.56	0.06	3.91	4.49	5.84	
Frisch = 0.75	1.03	1.08	1.49	0.07	3.94	4.42	5.82	
Autocorrelations	C	Y	n	NX	T	RER	e	
US data	0.84	0.85	0.91	0.86	0.83	0.82	0.82	
Infinite elasticity	0.66	0.68	0.68	0.68	0.57	0.65	0.72	
Frisch = 1.5	0.66	0.67	0.67	0.50	0.59	0.65	0.72	
Frisch = 0.75	0.65	0.66	0.66	0.55	0.59	0.64	0.72	
Cross-correlations	C,Y	Y,Y	n,Y	NX,Y	T,Y	RER,Y	e,Y	
US data	0.81	1.00	0.80	-0.39	0.09	0.11	0.09	
Infinite elasticity	0.91	1.00	0.85	-0.03	-0.40	0.43	0.39	
Frisch = 1.5	0.91	1.00	0.83	-0.23	-0.41	0.42	0.37	
Frisch = 0.75	0.90	1.00	0.82	-0.26	-0.41	0.41	0.36	
Sectoral standard deviations & cross-correlations	Y_{TH}^{Tot}	Y_N	n_{TH}	n_N	Y_{TH}^{Tot},Y	Y_N,Y	n_{TH},Y	n_N,Y
US data	2.50	0.50	1.98	0.89	0.90	0.49	0.82	0.74
Infinite elasticity	1.03	1.32	1.55	1.90	0.84	0.98	0.68	0.81
Frisch = 1.5	0.99	1.27	1.50	1.82	0.82	0.98	0.65	0.79
Frisch = 0.75	0.97	1.23	1.46	1.76	0.81	0.98	0.62	0.77

NOTE: The data moments have been computed using quarterly series for the period 1980:1 to 2007:4. Data sources and calculations are explained in the Appendix. All moments have been computed from logged and H-P-filtered series, except net exports, which are HP-filtered but not logged.

Table 3: Business cycle statistics under PCP

Standard deviations	C	Y	n	NX	T	RER	e	
US data	0.97	1.26	1.78	0.36	2.23	6.12	6.01	
Infinite elasticity	1.04	1.54	2.07	0.69	4.68	3.87	5.86	
Frisch = 1.5	1.03	1.42	1.90	0.55	3.18	3.95	5.85	
Frisch = 0.75	1.00	1.37	1.82	0.53	3.10	3.80	5.83	
Autocorrelations	C	Y	n	NX	T	RER	e	
US data	0.84	0.85	0.91	0.86	0.83	0.82	0.82	
Infinite elasticity	0.67	0.67	0.67	0.66	0.66	0.67	0.73	
Frisch = 1.5	0.66	0.64	0.65	0.58	0.51	0.67	0.73	
Frisch = 0.75	0.66	0.63	0.64	0.57	0.50	0.66	0.73	
Cross-correlations	C,Y	Y,Y	n,Y	NX,Y	T,Y	RER,Y	e,Y	
US data	0.81	1.00	0.80	-0.39	0.09	0.11	0.09	
Infinite elasticity	0.86	1.00	0.91	0.73	0.73	0.73	0.64	
Frisch = 1.5	0.88	1.00	0.89	0.69	0.66	0.69	0.58	
Frisch = 0.75	0.87	1.00	0.88	0.69	0.66	0.69	0.57	
Sectoral standard deviations & cross-correlations	Y_{TH}^{Tot}	Y_N	n_{TH}	n_N	Y_{TH}^{Tot},Y	Y_N,Y	n_{TH},Y	n_N,Y
US data	2.50	0.50	1.98	0.89	0.90	0.49	0.82	0.74
Infinite elasticity	2.00	1.45	2.65	2.04	0.92	0.97	0.86	0.83
Frisch = 1.5	1.88	1.33	2.49	1.89	0.91	0.97	0.84	0.80
Frisch = 0.75	1.82	1.29	2.41	1.83	0.90	0.97	0.83	0.79

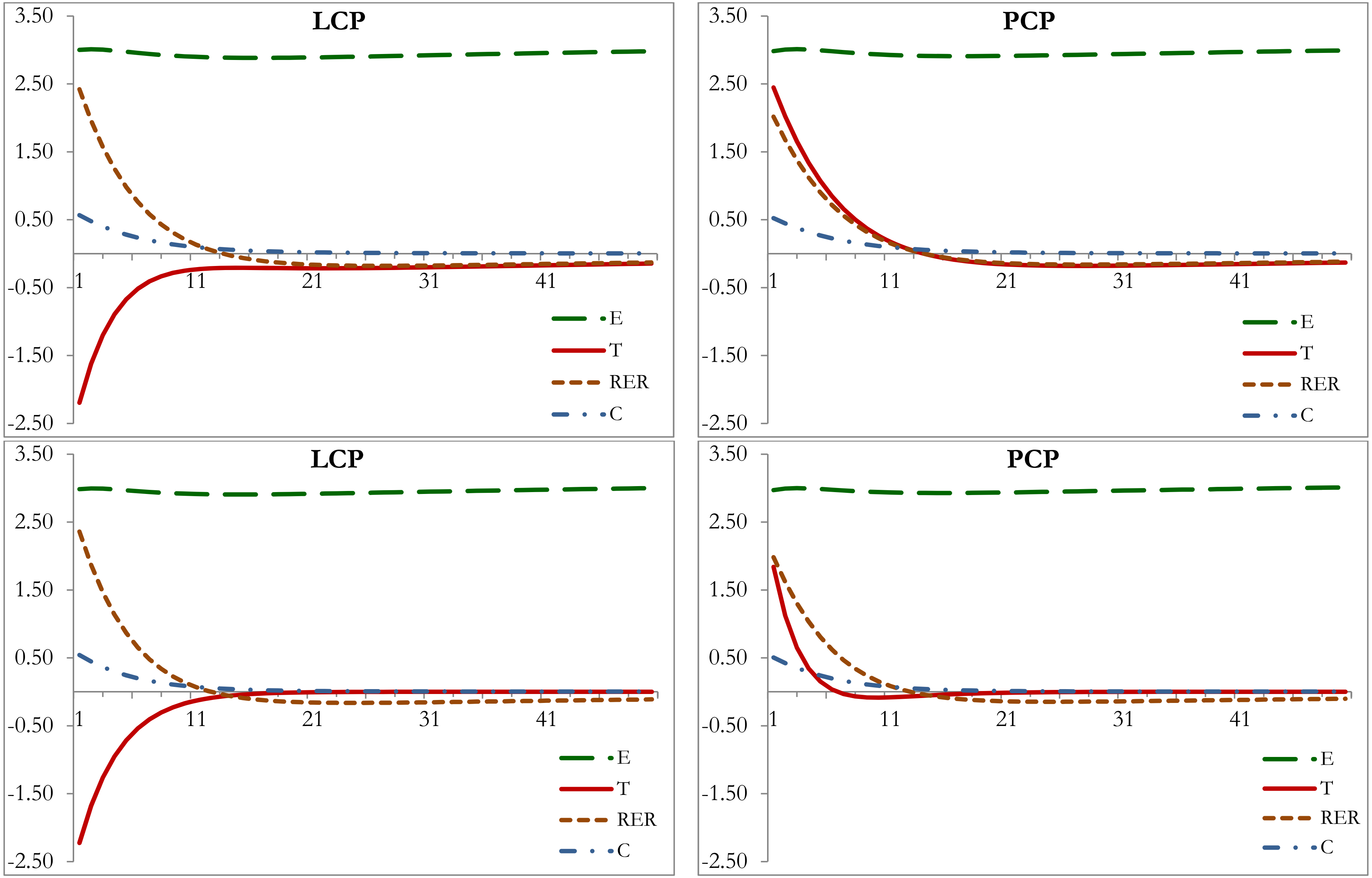
NOTE: See Table 2.

Table 4: Variance decompositions

Variables:	C	Y	n	NX	T	RER	e	Y_{TH}^{Tot}	Y_N	n_{TH}	n_N
Shocks:						LCP					
H money growth	82.93	45.11	32.71	37.85	43.40	48.77	49.99	28.29	34.91	31.09	27.78
F money growth	13.00	8.85	6.42	37.85	43.40	48.77	49.99	20.87	3.96	22.94	3.15
H tradeable technology	0.33	0.97	3.37	8.40	5.01	0.03	0.00	35.50	0.97	28.23	0.77
F tradeable technology	0.09	0.14	0.42	8.40	5.01	0.03	0.00	0.65	0.25	1.60	0.20
H nontradeable technology	3.41	1.72	25.01	3.52	1.48	1.15	0.01	13.07	7.22	14.36	25.71
F nontradeable technology	0.08	0.16	0.85	3.52	1.48	1.15	0.01	0.92	0.36	1.01	0.76
H government expenditure	0.16	43.05	31.22	0.22	0.11	0.05	0.00	0.68	52.31	0.75	41.62
F government expenditure	0.00	0.01	0.00	0.22	0.11	0.05	0.00	0.02	0.00	0.02	0.00
						PCP					
H money growth	80.32	62.11	49.08	47.78	45.24	48.40	49.99	64.86	40.66	67.86	32.92
F money growth	15.40	5.38	4.25	47.78	45.24	48.40	49.99	10.62	3.21	11.11	2.60
H tradeable technology	0.35	0.68	2.59	1.52	3.62	0.04	0.00	17.15	0.89	12.90	0.72
F tradeable technology	0.09	0.10	0.33	1.52	3.62	0.04	0.00	0.31	0.23	0.75	0.19
H nontradeable technology	3.58	1.24	19.09	0.66	1.05	1.49	0.01	6.25	6.65	6.54	23.97
F nontradeable technology	0.09	0.12	0.67	0.66	1.05	1.49	0.01	0.47	0.33	0.49	0.72
H government expenditure	0.17	30.36	23.99	0.04	0.08	0.06	0.00	0.33	48.03	0.34	38.89
F government expenditure	0.00	0.00	0.00	0.04	0.08	0.06	0.00	0.01	0.00	0.01	0.00

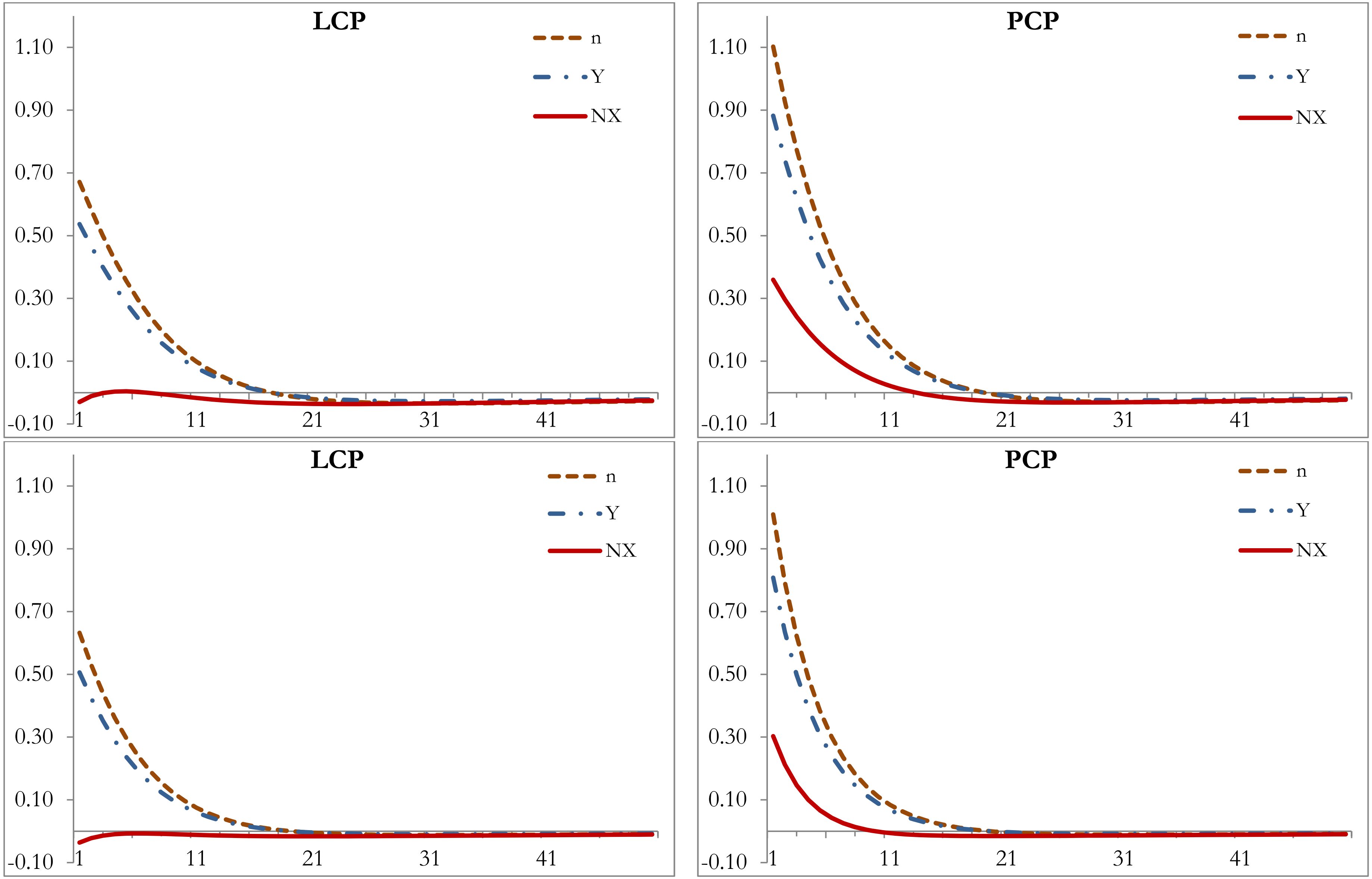
NOTE: Shocks are orthogonalised using the Cholesky method, and the horizon is set at 200 quarters. Each column reports, for each variable, the share of the total variance explained by every shock, measured in per cent. The numbers are averages across all possible variance decompositions, given by the number of different orderings of the 8 shocks (40,320).

Figure 1: Impulse responses of consumption, terms of trade and real and nominal exchange rates to a 1% Home monetary shock. Infinite Frisch elasticity (top) and Frisch = 0.75 (bottom).



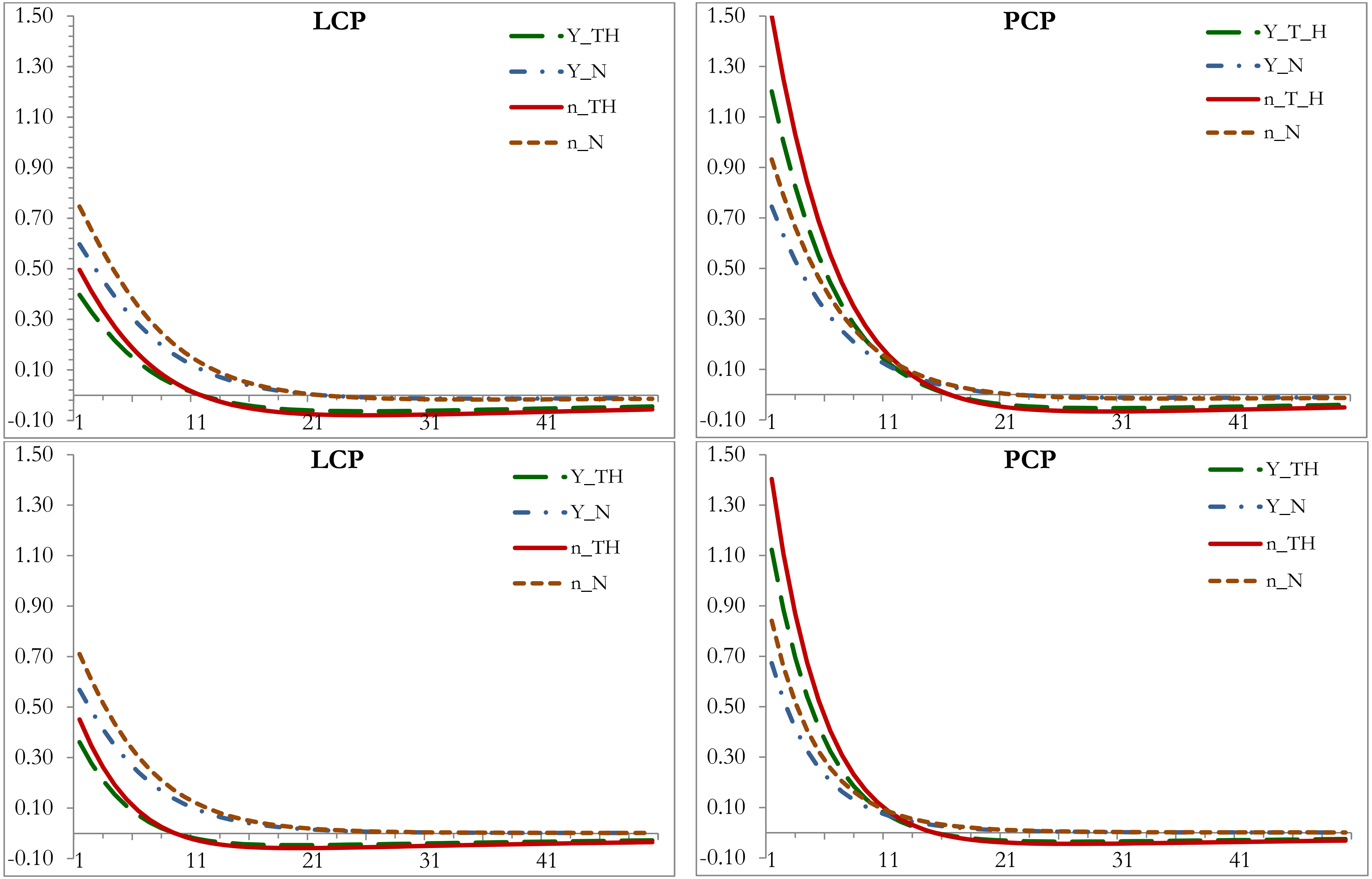
Note: Time is in quarters.

Figure 2: Impulse responses of output, net exports and the labour input to a 1% Home monetary shock. Infinite Frisch elasticity (top) and Frisch = 0.75 (bottom).



Note: Time is in quarters.

Figure 3: Impulse responses of sectoral output and labour inputs to a 1% Home monetary shock. Infinite Frisch elasticity (top) and Frisch = 0.75 (bottom).



Note: Time is in quarters.

Appendices

A.1 Data sources and calculations

Alias	*Description*	*Source*[a]
	Exports of goods and services; Imports of goods and services (chained volume estimates)	OECD QNA
	Exports deflator; Imports deflator	OECD QNA
	Dollar exchange rates	IMF IFS
C	Private final consumption expenditure	OECD QNA
Y	Gross Domestic Product	OECD QNA
n	Aggregate weekly hours index, total private industries (quarterly averages of monthly data)	FRED
NX	Exports - Imports of goods and services /GDP	
T	Exports deflator / Imports deflator	
e	Geometric GDP-weighted average of France, Germany, Canada, Japan, Mexico and UK dollar exchange rates	
P	Consumer Price Index for all items	OECD MEI
P^*	Geometric GDP-weighted average of Canada, France, Germany, Japan, Mexico and UK CPI indexes	
RER	$= eP^*/P$	
Y_{TH}^{Tot}	Index of production in total manufacturing	OECD MEI
Y_N	Index of real Gross Domestic Product of services	BEA NIPA
n_{TH}	Employees in manufacturing	OECD MEI
n_N	Employees of service-providing industries (quarterly averages of monthly data)	BLS

[a] Legend: BEA NIPA = Bureau of Economic Analysis, National Income and Product Accounts; BLS = Bureau of Labor Statistics; FRED = Federal Reserve Economic Data; IMF IFS = IMF International Financial Statistics; OECD MEI = OECD Main Economic Indicators; OECD QNA = OECD Quarterly National Accounts.

A.2 Nonconvexity and individual heterogeneity

In this Appendix I analyse the relationship between the nonconvexity of the individual commodity set and the labour supply elasticity. Do employment lotteries always result in an infinite labour supply elasticity? Or is the infinite elasticity due to the homogeneity of individuals' preferences? I will answer these questions first in the context of the indivisible labour model (i.e. when individuals are not able to adjust the number of hours worked), since the literature to date has focused on this type of nonconvexity. Then I will turn to the model presented in this paper, in which I assume that individuals can adjust the number of hours worked, but their commodity set is nonconvex because they cannot work in two sectors at the same time.

Some authors have shown that indivisible labour is not by itself a sufficient condition for the aggregate labour supply to have infinite elasticity. This point is made, for example, by Christiano, Trabandt and Walentin (2010), and is based on a model where individuals differ in their preference for leisure (or aversion to work). By assumption the economy is populated by a continuum of individuals, indexed with $i \in [0, 1]$. The utility of individual i is given by:

$$\log(C_t) - i^{\nu},\ \nu > 0\ , \tag{1}$$

if employed, and by:

$$\log(C_t)\ \ , \tag{2}$$

if unemployed.

Individuals are ranked according to their degree of aversion to work. Those with high i have a strong aversion to work, and those with low i have a low aversion to work. If n_t is employment, then those with $0 \leq i \leq n_t$ work and those with $i > n_t$ do not. Individuals either work some fixed workweek or not at all.[1] Everyone receives the same level of consumption. Aggregate utility is given by:

$$\int_0^1 \log\left(C_t\right)\, di - \int_0^{n_t} i^{\nu}\, di = \log\left(C_t\right) - \frac{n_t^{\nu+1}}{\nu+1}\ . \tag{3}$$

According to (3) the aggregate labour supply elasticity is equal to $1/\nu$, hence it is possible to have a finite labour supply elasticity in an indivisible labour environment if individuals are heterogeneous. Moreover any finite aggregate labour supply elasticity can be calibrated by making an assumption on the cross-sectional distribution of skills or taste parameters.

However, Christiano, Trabandt and Walentin's (2010) result depends on a particular aggregation method, one which assumes that individuals are ranked according to their aversion to work, so that only those with low aversion go to work, while others never go to work (as long as $n_t < 1$). One must find a justification for why individuals would spontaneously choose such arrangement, or alternatively, one could justify the aggregation of preferences (3) by means of a social planner.[2] Equation (3) is the social planner's objective function, and

[1] This assumption justifies why the amount of time spent at work does not affect the preference ordering.

[2] Christiano, Trabandt and Walentin (2010) assume instead a benevolent household, which behaves as a *de facto* social planner.

the welfare-maximising equilibrium is the one in which only those individuals with a low aversion to work are employed.[3]

Aggregate outcomes are different in a competitive equilibrium with lotteries. In this set-up $n_t(i)$ is the probability of being employed, and consistently with expected utility theory it enters the utility of individual i linearly:

$$\log[C_t(i)] - n_t(i) \cdot i^{\nu} \ . \tag{4}$$

The individual i's choice of employment lottery must satisfy the following first-order condition:

$$\frac{1}{C_t(i)} \frac{W_t}{P_t} = i^{\nu} \ , \tag{5}$$

where P_t is the aggregate price index, and W_t is the market price of labour, assumed to be the same for all individuals.[4] Let the aggregate labour supply be defined as:

$$n_t \equiv \int_0^1 n_t(i) \ di \ . \tag{6}$$

Notice that (5) and (6) imply that the aggregate labour supply elasticity is infinite. Therefore, heterogeneity *per se* does not guarantee a finite elasticity of labour supply, and much depends on how the aggregate variables are

[3] The social planner solution also emerges in an economy where the individuals commit to a risk-sharing arrangement. This solution concept is applied by Janko (2011), who assumes non-separability in consumption and leisure and homogeneous preferences.

[4] This assumption helps to simplify the analysis but is not crucial.

derived from the preferences of heterogeneous individuals. In a competitive equilibrium with indivisible labour and employment lotteries the elasticity of labour supply is infinite, even with agent heterogeneity.

On a side note, notice that this result depends on the assumption that individuals know their type i when they solve the maximisation problem, and the alternative assumption that individuals do not know their type would bring about a totally different result. In such alternative scenario, equation (5) would hold in expectation and everyone would choose the same $C_t(i)$ and $n_t(i)$. *Ex-post*, after types are revealed, an individual who is allowed to re-optimise while keeping the same $C_t(i)$ would choose $n_t(i) = 0$. Again in this scenario individuals would not choose an equilibrium in which only those with low i work.

I now turn to the model of Section 2 to further investigate the relationship between nonconvexity and the labour supply elasticity. Here nonconvexity arises because individuals cannot work in two sectors at the same time. I modify the utility function by assuming that individuals are heterogeneous in their preference for leisure, in a way similar to Christiano, Trabandt and Walentin (2010), but I keep the same functional form as in Section 2. For the sake of simplicity, I consider a one-period economy with no money and no bonds, but all the other assumptions remain unchanged.

The Home country is populated by a continuum of individuals $i \in (0, 1]$ who differ in regard to their preference for leisure. The utility function of individual i is given by:

$$\frac{C^{1-\sigma}-1}{1-\sigma}+\upsilon\left(\mathbf{h}_{TH},\,\mathbf{h}_{N},\,i\right)\ , \tag{7}$$

$$\upsilon\left(\mathbf{h}_{TH},\,\mathbf{h}_{N},\,i\right)=\begin{cases}\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{TH}\right)^{\omega}i^{\nu} & \text{if } \mathbf{h}_{TH}\neq 0\ ,\\ \frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{N}\right)^{\omega}i^{\nu} & \text{if } \mathbf{h}_{N}\neq 0\ ,\\ \frac{\kappa}{\omega}\left(\tau\right)^{\omega}i^{\nu} & \text{if } \mathbf{h}_{TH}=\mathbf{h}_{N}=0\ ,\end{cases}$$

with $\omega, \nu > 0$.

Social planner problem

A social planner assigns a measure n_{TH} of individuals to the tradeable goods sector and a measure n_N to the nontradeable goods sector. Those employed in the tradeable goods sector supply $\mathbf{h}_{TH}$ hours and those employed in the nontradeable goods sector supply $\mathbf{h}_N$ hours of work. All individuals working in a sector work the same hours, however both $\mathbf{h}_{TH}$ and $\mathbf{h}_N$ are choice variables. As in Christiano, Trabandt and Walentin (2010) (and the literature on nonconvexity in labour supply) I assume that the social planner gives everyone the same level of consumption.[5] I assume that profits from monopolistically competitive firms are distributed equally.

The utilitarian planner allocates the individuals with the lowest i to the sector with the longer workweek. For example, if $\tau \geq 1-\psi-\mathbf{h}_N \geq 1-\psi-\mathbf{h}_{TH}$ then the social planner's objective is to maximise:

[5] Thus the allocation of consumption is the same as in the competitive equilibrium with lotteries which I will discuss later.

$$\max \quad \frac{C^{1-\sigma}-1}{1-\sigma} + \int_0^{n_{TH}} \frac{\kappa}{\omega} \left(1-\psi-\mathbf{h}_{TH}\right)^{\omega} i^{\nu}\, di$$
$$+ \int_{n_{TH}}^{n_{TH}+n_N} \frac{\kappa}{\omega} \left(1-\psi-\mathbf{h}_{N}\right)^{\omega} i^{\nu}\, di + \int_{n_{TH}+n_N}^{1} \frac{\kappa}{\omega} \left(\tau\right)^{\omega} i^{\nu}\, di \ , \qquad (8)$$

subject to:

$$PC \le n_{TH} W_{TH} \mathbf{h}_{TH} + n_N W_N \mathbf{h}_N + \int_0^1 \Pi_{TH}\left(f_{TH}\right) df_{TH} + \int_0^1 \Pi_N\left(f_N\right) df_N \ . \qquad (9)$$

The first-order conditions with respect to hours and participation rates are:

$$\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{TH}\right)^{\omega} n_{TH}^{\nu} + \frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{N}\right)^{\omega}\left(n_{TH}+n_N\right)^{\nu}$$
$$-\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{N}\right)^{\omega} n_{TH}^{\nu} - \frac{\kappa}{\omega}\left(\tau\right)^{\omega}\left(n_{TH}+n_N\right)^{\nu} = -\frac{C^{-\sigma}}{P} W_{TH}\mathbf{h}_{TH} \ , \qquad (10)$$

$$\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{N}\right)^{\omega}\left(n_{TH}+n_N\right)^{\nu} - \frac{\kappa}{\omega}\left(\tau\right)^{\omega}\left(n_{TH}+n_N\right)^{\nu} = -\frac{C^{-\sigma}}{P} W_N \mathbf{h}_N \ , \qquad (11)$$

$$\kappa\left(1-\psi-\mathbf{h}_{TH}\right)^{\omega-1} \frac{\left(n_{TH}\right)^{\nu+1}}{\nu+1} = -\frac{C^{-\sigma}}{P} n_{TH} W_{TH} \ , \qquad (12)$$

$$\kappa\left(1-\psi-\mathbf{h}_N\right)^{\omega-1}\left[\frac{\left(n_{TH}+n_N\right)^{\nu+1}}{\nu+1}-\frac{\left(n_{TH}\right)^{\nu+1}}{\nu+1}\right]=-\frac{C^{-\sigma}}{P}n_N W_N\ . \quad (13)$$

Equations (10) to (13) show that in the social planner solution participation rates do not enter linearly the first-order conditions. Hence, it is possible to have a finite labour supply elasticity in the model if individuals are heterogeneous. The parameter ν can be used to calibrate the labour supply elasticity in a given parameterization.

Competitive equilibrium with lotteries and insurance market

An individual chooses a probability $n_{TH}(i)$ of working in the tradeable sector and a probability $n_N(i)$ of working in the nontradeable sector. A lottery is held to determine which individuals must work and in which sector. Individuals are paid only for the work that they actually do, but have access to an insurance market. Because there are two sources of income risk, the risk of being unemployed and the risk of being allocated to the sector paying the lowest wage, one insurance contract is not enough to attain full insurance. Therefore I assume that individuals buy two policies with two separate insurance firms. Under policy 1 a premium is due if employed in sector TH, and under policy 2 a premium is due if employed in sector N. Both policies pay out a compensation in case of unemployment. I now show that this arrangement is sufficient to deliver full insurance.

The individual i chooses the compensation levels $y_1(i)$ and $y_2(i)$ by solving the following problem:

$$\max \quad n_{TH}(i)\left[\frac{(C(i|TH))^{1-\sigma}-1}{1-\sigma}+\frac{\kappa}{\omega}(1-\psi-\mathbf{h}_{TH}(i))^{\omega}i^{\nu}\right]$$
$$+n_N(i)\left[\frac{(C(i|N))^{1-\sigma}-1}{1-\sigma}+\frac{\kappa}{\omega}(1-\psi-\mathbf{h}_{N}(i))^{\omega}i^{\nu}\right]$$
$$+(1-n_{TH}(i)-n_N(i))\left[\frac{(C(i|U))^{1-\sigma}-1}{1-\sigma}+\frac{\kappa}{\omega}(\tau)^{\omega}i^{\nu}\right] , \tag{14}$$

subject to:

$$PC(i|TH)\leq W_{TH}\mathbf{h}_{TH}(i)+\int_0^1\Pi_{TH}(f_{TH})\,df_{TH}+\int_0^1\Pi_N(f_N)\,df_N-p_1(i)\,y_1(i) , \tag{15}$$

$$PC(i|N)\leq W_{N}\mathbf{h}_{N}(i)+\int_0^1\Pi_{TH}(f_{TH})\,df_{TH}+\int_0^1\Pi_N(f_N)\,df_N-p_2(i)\,y_2(i) , \tag{16}$$

$$PC(i|U)\leq \int_0^1\Pi_{TH}(f_{TH})\,df_{TH}+\int_0^1\Pi_N(f_N)\,df_N+y_1(i)+y_2(i) , \tag{17}$$

where $C(i|TH)$, $C(i|N)$ and $C(i|U)$ are consumption of individual i contingent on working in sectors TH, N or being unemployed, and $p_1(i)$ and $p_2(i)$ are the two insurance prices.

The first-order conditions with respect to $y_1(i)$ and $y_2(i)$ are:

$$n_{TH}(i)\left(C(i|TH)\right)^{-\sigma} p_1(i) = \left(1 - n_{TH}(i) - n_N(i)\right)\left(C(i|U)\right)^{-\sigma} \ , \tag{18}$$

$$n_N(i)\left(C(i|N)\right)^{-\sigma} p_2(i) = \left(1 - n_{TH}(i) - n_N(i)\right)\left(C(i|U)\right)^{-\sigma} \ . \tag{19}$$

Expected profits of both insurance firms are zero:

$$n_{TH}(i)\, p_1(i)\, y_1(i) - \left(1 - n_{TH}(i) - n_N(i)\right) y_1(i) = 0 \ , \tag{20}$$

$$n_N(i)\, p_2(i)\, y_2(i) - \left(1 - n_{TH}(i) - n_N(i)\right) y_2(i) = 0 \ , \tag{21}$$

therefore:

$$p_1(i) = \frac{1 - n_{TH}(i) - n_N(i)}{n_{TH}(i)} \ , \tag{22}$$

$$p_2(i) = \frac{1 - n_{TH}(i) - n_N(i)}{n_N(i)} \ . \tag{23}$$

Substituting (22) and (23) into the first-order conditions (18) and (19) we obtain:

$$n_{TH}(i)\left(C(i|TH)\right)^{-\sigma}\frac{1-n_{TH}(i)-n_N(i)}{n_{TH}(i)}=\left(1-n_{TH}(i)-n_N(i)\right)\left(C(i|U)\right)^{-\sigma} \ , \tag{24}$$

$$n_N(i)\left(C(i|N)\right)^{-\sigma}\frac{1-n_{TH}(i)-n_N(i)}{n_N(i)}=\left(1-n_{TH}(i)-n_N(i)\right)\left(C(i|U)\right)^{-\sigma} \ , \tag{25}$$

which show that $C(i|TH)=C(i|N)=C(i|U)$. Therefore, the individual insures herself fully against income risk. Consumption of individual i, denoted by $C(i)$ from now on, is independent of the employment status. Moreover, since the left-hand sides of the constraints (15), (16) and (17) are identical, $y_1(i)$ and $y_2(i)$ are chosen so that the right-hand sides are identical too. This implies that income is equal to the expected wage given the lottery, regardless of the sector of employment:

$$PC(i)\leq n_{TH}(i)W_{TH}\mathbf{h}_{TH}(i)+n_N(i)W_N\mathbf{h}_N(i)+\int_0^1\Pi_{TH}(f_{TH})\,df_{TH}+\int_0^1\Pi_N(f_N)\,df_N \ . \tag{26}$$

The optimal choice of hours $\mathbf{h}_{TH}(i)$ and $\mathbf{h}_N(i)$, and probabilities $n_{TH}(i)$ and $n_N(i)$ satisfies the first-order conditions of the following problem:

$$\max \quad \frac{C(i)^{1-\sigma}-1}{1-\sigma}+n_{TH}(i)\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{TH}(i)\right)^{\omega}i^{\nu}$$
$$+n_{N}(i)\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{N}(i)\right)^{\omega}i^{\nu}+\left(1-n_{TH}(i)-n_{N}(i)\right)\frac{\kappa}{\omega}(\tau)^{\omega}i^{\nu}, \quad (27)$$

subject to (26). These first-order conditions are:

$$\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{TH}(i)\right)^{\omega}i^{\nu}-\frac{\kappa}{\omega}(\tau)^{\omega}i^{\nu}+C(i)^{-\sigma}\frac{W_{TH}\mathbf{h}_{TH}(i)}{P}=0\,, \qquad (28)$$

$$\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{N}(i)\right)^{\omega}i^{\nu}-\frac{\kappa}{\omega}(\tau)^{\omega}i^{\nu}+C(i)^{-\sigma}\frac{W_{N}\mathbf{h}_{N}(i)}{P}=0\,, \qquad (29)$$

$$n_{TH}(i)\,\kappa\left(1-\psi-\mathbf{h}_{TH}(i)\right)^{\omega-1}i^{\nu}+C(i)^{-\sigma}\frac{n_{TH}(i)\,W_{TH}}{P}=0\,, \qquad (30)$$

$$n_{N}(i)\,\kappa\left(1-\psi-\mathbf{h}_{N}(i)\right)^{\omega-1}i^{\nu}+C(i)^{-\sigma}\frac{n_{N}(i)\,W_{N}}{P}=0\,, \qquad (31)$$

which in turn imply:

$$\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_{TH}(i)\right)^{\omega}-\kappa\left(1-\psi-\mathbf{h}_{TH}(i)\right)^{\omega-1}\mathbf{h}_{TH}(i)=\frac{\kappa}{\omega}(\tau)^{\omega}\,, \qquad (32)$$

$$\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}_N\left(i\right)\right)^{\omega}-\kappa\left(1-\psi-\mathbf{h}_N\left(i\right)\right)^{\omega-1}\mathbf{h}_N\left(i\right)=\frac{\kappa}{\omega}\left(\tau\right)^{\omega}\ . \tag{33}$$

Therefore, $\mathbf{h}_{TH}\left(i\right)=\mathbf{h}_N\left(i\right)\equiv\mathbf{h}$ for all i, and $W_{TH}=W_N\equiv W$. Consequently the four first-order conditions reduce to just two:

$$\frac{\kappa}{\omega}\left(1-\psi-\mathbf{h}\right)^{\omega}i^{\nu}-\frac{\kappa}{\omega}\left(\tau\right)^{\omega}i^{\nu}+C\left(i\right)^{-\sigma}\frac{W}{P}\mathbf{h}=0\ , \tag{34}$$

$$\kappa\left(1-\psi-\mathbf{h}\right)^{\omega-1}i^{\nu}+C\left(i\right)^{-\sigma}\frac{W}{P}=0\ . \tag{35}$$

Equations (34) and (35) imply that optimal hours are the same for each individual and do not depend on $\frac{W}{P}$. However, heterogeneous individuals choose different probabilities, therefore the law of large numbers is not applicable.

Let us define $n\left(i\right)\equiv n_{TH}\left(i\right)+n_N\left(i\right)$ and $n\equiv\int_0^1 n\left(i\right)\ di$. It is easy to verify that the elasticity of aggregate labour supply:

$$\frac{\vartheta\left(n\mathbf{h}\right)}{\vartheta W/P}\frac{W/P}{n\mathbf{h}}=\frac{\vartheta n}{\vartheta W/P}\frac{W/P}{n}\ ,$$

is infinite. Therefore, similarly to the indivisible labour model, individual heterogeneity in itself does not guarantee that the labour supply has finite elasticity. Whether or not this is the case depends on the choice of equilibrium and how the aggregate variables are derived from the preferences of heterogeneous individuals.