Are the New British Universities Congested?

A.T. Flegg and D.O. Allen*

ABSTRACT

This paper uses data envelopment analysis (DEA) to examine the issue of congestion in British universities. The focus of the paper is on 41 former polytechnics that became universities in 1992, and the analysis covers the period 1995/6 to 2003/4. These new universities differ from the older universities in many ways, especially in terms of their far higher student : staff ratios and substantially lower research funding per member of staff. The primary aim of the paper is to examine whether this under-resourcing of the new universities has led to 'congestion', in the sense that their output has been reduced as a result of having too many students. Three alternative methods of measuring congestion are examined and, to check the sensitivity of the results to different specifications, three alternative DEA models are formulated. The results reveal that a substantial amount of congestion was present throughout the period under review, and in a wide range of universities, but whether it rose or fell is uncertain, as this depends on which congestion model is used. The results indicate that an overabundance of undergraduate students was the largest single cause of congestion in the former polytechnics during the period under review. Less plausibly, the results also suggest that academic overstaffing was a major cause of congestion! By contrast, postgraduates and 'other expenditure' are found to play a noticeably smaller role in generating congestion.

KEY WORDS: British universities; congestion; DEA

1. INTRODUCTION

Higher education in the United Kingdom has expanded rapidly in recent years, continuing a process that began in the 1960s. This growth has occurred in the 45 older universities (those existing prior to 1992), as well as in other higher education institutions. The latter include the

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* Tony.Flegg@uwe.ac.uk and David.Allen@uwe.ac.uk. School of Economics, Bristol Business School, University of the West of England, Coldharbour Lane, Bristol BS16 1QY.

former polytechnics that became universities in 1992, university colleges, institutes of higher education and so on. Here we have chosen to look at the experience of the former polytechnics in the period 1995/6 to 2003/4. These institutions form a relatively homogeneous group, sharing a common history and facing similar opportunities and problems. As far as we are aware, this is the first study to employ data envelopment analysis (DEA) to examine the efficiency of the former polytechnics as a separate group.[1]

Figure 1 near here

Figure 1 illustrates the point that the former polytechnics operate under much higher student : staff ratios than do the older universities. In addition, the older universities typically receive substantially more research funding per member of staff, and a higher proportion of their undergraduate students gain first-class degrees and upper seconds.[2] It is also interesting to observe from Figure 1 that there has been a much smaller rise in the number of students in the former polytechnics than in the older universities.[3] In view of these clear disparities, it seems appropriate to analyse the older universities and the former polytechnics separately.[4]

2. THE PROBLEM OF CONGESTION

The focus of this paper is on the problem of congestion, which refers to a situation where the use of a particular input has increased by so much that output actually falls. The specific issue we wish to explore here is whether the number of students in the former polytechnics has grown to such an extent that it has caused in output to fall.

Cooper *et al.* (2001a, p. 62) define congestion in the following way:

Definition 1. *Input congestion occurs whenever increasing one or more inputs decreases some outputs without improving other inputs or outputs. Conversely, congestion occurs when decreasing some inputs increases some outputs without worsening other inputs or outputs.*

They go on to observe (*ibid.*, p. 63) that congestion can be regarded as a particularly severe form of technical inefficiency.

However, the above definition makes no reference to any limiting factor that might account for the congestion. A possible alternative definition might read as follows:

Definition 2. *Input congestion occurs whenever more (less) of any input is employed, with all other inputs held constant, and this leads to a fall (rise) in output*. This alternative definition is grounded in the hypothesis of diminishing marginal returns, with the added feature that congestion requires a negative marginal product to occur eventually.

In the case of universities, it seems reasonable to assume that an excessive number of students could lead to congestion. For instance, Figure 1 shows that the number of full-time equivalent students in the former polytechnics increased substantially in the period under review; as a result, the marginal product of students might have become *negative* in some universities. The implication of this is that a reduction in the number of students, with all other inputs (staff, buildings, etc.) held constant, would raise the university's output in terms of research and degrees awarded, both undergraduate and postgraduate. On the other hand, Figure 1 shows that this expansion was accompanied by only a modest rise in the student : staff ratio for the period as a whole.[5]

3. DEA MODELS

DEA makes use of linear programming techniques to construct an 'efficiency frontier', with the most efficient organizations within a group being used to define the standard against which the performance of the other organizations is evaluated. The concept of efficiency is thus relative rather than absolute. The organizations being evaluated are known as decision-making units (DMUs).

The starting point for our analysis is the Charnes–Cooper–Rhodes (CCR) model, which assumes constant returns to scale (CRS) and no congestion. In its output-oriented form, this model can be specified as follows:

$$\theta^* = \max \theta \tag{1a}$$

subject to:

$$\sum_j \lambda_j x_{ij} \leq x_{ik} \qquad i = 1, 2, \ldots, m \tag{1b}$$

$$\sum_j \lambda_j y_{rj} \geq \theta y_{rk} \qquad r = 1, 2, \ldots, s \tag{1c}$$

$$\lambda_j \geq 0 \qquad j = 1, 2, \ldots, n \tag{1d}$$

where x_{ij} and y_{rj} are the quantities of input *i* and output *r* produced by DMU *j*, and the λ_j are a set of weights with values to be determined. The model is solved for each DMU *k*, and an efficiency score, $\theta^* \geq 1$, is thereby produced. It is more convenient, however, to define a new measure of *technical efficiency*, $TE \equiv 1/\theta^*$, so that efficient DMUs have TE = 1, whereas inefficient DMUs have TE < 1.

Figure 2 near here

This model is illustrated in Figure 2.[6] For simplicity, it is assumed that each DMU employs a single input, x, to produce a single output, y. DMUs B and C operate under CRS and hence are located on the CCR frontier; both have TE = 1. The other DMUs are deemed to be inefficient. For example, A has TE = 0.5, showing that it is producing only half of its potential output; to be efficient, it would need to move to point A´ on the frontier.

To capture possible *scale* effects, we need to modify the CCR model to produce the following Banker–Charnes–Cooper (BCC) model.[7]

$$\phi^* = \max \phi \tag{2a}$$

subject to:

$$\sum_j \lambda_j x_{ij} \leq x_{ik} \qquad i = 1, 2, \ldots, m \tag{2b}$$

$$\sum_j \lambda_j y_{rj} \geq \phi y_{rk} \qquad r = 1, 2, \ldots, s \tag{2c}$$

$$\sum_j \lambda_j = 1 \tag{2d}$$

$$\lambda_j \geq 0 \qquad j = 1, 2, \ldots, n \tag{2e}$$

The crucial difference between these two models is the addition of the convexity constraint $\sum_j \lambda_j = 1$. In Figure 2, this constraint generates a new frontier ABCDE and its horizontal extension from E. This BCC frontier exhibits variable returns to scale (VRS). The BCC model is solved in two stages. In the first stage, ϕ^* is evaluated for each DMU *k*, while the second stage involves maximizing the sum of the slacks, conditional on this value of ϕ^* (cf. Cooper *et al.*, 2000b, pp. 3–5).

In terms of the new model, A and D are now regarded as being efficient. However, even though E has $\phi^* = 1$, it is still deemed to be inefficient owing to the *slack* of one unit in x. Notice that x can be reduced by one unit without affecting y. Identifying inefficiencies of this kind is the aim of the second stage of the BCC model.

To measure *scale efficiency*, we can define a new ratio, $SE \equiv \phi^*/\theta^*$. This yields SE = 1 for B and C but values of 0.5 for A, 0.833 for D and 0.714 for E. The diagram shows that A is subject to increasing returns to scale, whereas D and E are subject to decreasing returns. What is more, all of the inefficiency of these three DMUs can be attributed to the fact that they are operating at an inappropriate scale.

As regards F and G, it is clear from Definition 2 above that both DMUs would be regarded as being congested. This is because y and x are inversely related over the relevant part of the frontier. By contrast, E would be held to be technically inefficient rather than congested. This is because y is constant over the range x = 6 to x = 7. Classifying the remaining three DMUs is a little more complicated but their situation becomes clearer once we project them onto the frontier: J to D, I to E and H to F. Once this is done, it is evident that only H suffers from congestion. Even so, all three DMUs do suffer from *pure technical* inefficiency. This is because they are located beneath the frontier ABCDEFG.

4. MEASURING CONGESTION

The conventional way of measuring congestion was developed by Färe and Grosskopf, while Byrnes *et al.* (1984) and Färe *et al.* (1985a) were the first published applications. Cooper *et* *al.* (1996) then proposed an alternative procedure, which was refined and applied to Chinese data by Brockett *et al.* (1998) and Cooper *et al.* (2000b). More recently, Tone and Sahoo (2004) have proposed a new approach to measuring congestion. For ease of exposition, these alternative procedures are referred to hereafter as the approaches of Färe, Cooper and Tone.

The theoretical merits and demerits of the competing approaches of Cooper and Färe have been debated most recently by Cherchye *et al.* (2001) and Cooper *et al.* (2001a, b), yet this debate was inconclusive. There is also little published information on whether these two approaches yield very different outcomes in terms of the measured amount of congestion. Hence it is important to consider carefully which approach or approaches to pursue.

An important consideration is the orientation of the model. Here we would argue that an objective of maximizing output from given resources is likely to be much closer to the aims of British universities than the alternative of minimizing the resources used to produce a given output. In addition, we would maintain that the problem of congestion in British universities, if it exists, is likely to be one of excessive inputs.

However, in the current version of *OnFront*, the software supporting Färe's approach, congestion of inputs is measured using an input-oriented approach, whereas congestion of outputs is captured via an output-oriented approach.[8] In the case of outputs, congestion refers to a situation where one or more of the outputs is an undesirable by-product of joint production, e.g. air pollution associated with the generation of electricity (cf. Färe *et al.*, 1989). Since all three outputs in our model are deemed to be desirable, congestion of outputs can be ruled out *a priori*. On the other hand, there are sound reasons for anticipating congestion with respect to one or more of the inputs.

Thus a disadvantage of using Färe's approach in the present context is that it would entail adopting an input-oriented rather than an output-oriented approach. By contrast, the approaches of Cooper and Tone permit one to measure congestion of inputs via an output-oriented approach; they are, therefore, preferable in this respect. Moreover, we would argue that Färe's approach has a serious shortcoming when compared with those of Cooper and Tone: only certain instances of negative marginal productivity are deemed to constitute congestion and these cases may not even be the most plausible ones (see Flegg and Allen, 2006). Tone's approach also has the advantage that one can obtain a measure of the extent of the scale diseconomies affecting individual universities.

The most attractive feature of Cooper's approach is that it makes use of concepts that can easily be identified and measured in a set of data. What is more, his measure of congestion is easy to understand and one can immediately see which factors are apparently causing the problem and to what extent. This is more difficult to establish from Färe's procedure (see Cooper *et al.*, 2000b, pp. 6–7). However, a demerit of Cooper's non-radial methodology, in comparison with Färe's radial approach, is that a straightforward decomposition of overall technical efficiency into scale, congestion and purely technical components cannot be carried out. It is also not entirely clear what aspects of the data Cooper's formula is trying to capture: is it negative marginal productivity or severe scale diseconomies or both?

In the light of the above discussion, we shall be using the approaches of Cooper and Tone as the basis for our measurements of congestion. However, later in the paper, we shall use Färe's approach to assess the sensitivity of the findings to changes in the underlying technology, i.e. to see what difference it makes if we assume constant rather than variable returns to scale.

5. COOPER'S MEASURE OF CONGESTION

Cooper's measure of congestion, denoted here by C_C, is calculated from the results of the BCC model. It involves a straightforward decomposition of the slacks from this model. At the outset, Cooper *et al.* (2001a, p. 69) posit a relationship of the following form:

$$c_i = s_i^* - \delta_i^* \qquad (3)$$

where c_i is the amount of congestion associated with input i, s_i^* is the total amount of slack in input i and δ_i^* is the amount of slack attributable to technical inefficiency. The measured amount of congestion is thus a residual derived from the DEA results. The next step is to rewrite equation (3) as follows:

$$c_i/x_i = s_i^*/x_i - \delta_i^*/x_i \tag{4}$$

where c_i/x_i is the proportion of congestion in input i, s_i^*/x_i is the proportion of slack in input i and δ_i^*/x_i is the proportion of technical inefficiency in input i. The final step is to take arithmetic means over all inputs to get:

$$C_C = \overline{s/x} - \overline{\delta/x} \tag{5}$$

Hence C_C measures the average proportion of congestion in the inputs used by a particular DMU. It has the property $0 \leq C_C \leq 1$. See Cooper *et al.* (2001a, p. 73).

To illustrate the meaning of Cooper's measure, let us return to Figure 2. It was noted earlier that DMUs F and G were both congested but we now need to measure the extent of this congestion. G will be examined first. The diagram reveals that there are two DMUs that could be used for evaluating G, viz D and E. However, although both would yield $\phi^* = 2.5$, D is the one that would maximize the slack in input x (giving three units rather than two). Hence D is the DMU picked out by Cooper's model for the purpose of evaluating G. In this instance, the three units of slack in input x obtained from the BCC model would be divided into two units of congestion and one unit of technical inefficiency. In terms of equation (5), we would have $\overline{s/x} = 3/9$ and $\overline{\delta/x} = 1/9$, giving $C_C = 2/9 = 0.222$. Similarly, we can calculate $C_C = (2/8 - 1/8) = 0.125$ for F (and likewise for H). I and J would be free from congestion.

It is worth noting that, in real data sets, horizontal segments such as DE in Figure 2 are rare and, in our own data set of 41 universities over nine years, we found no instance where slack existed, yet $\phi^* = 1$. If the data set does not have any DMUs like E, then the amount of congestion for each input equals the BCC slack for this input. This greatly simplifies the work needed to compute C_C.

6. A NUMERICAL EXAMPLE

To clarify the meaning of Cooper's measure, consider Figure 3.[9] This shows six hypothetical universities. Whereas university M produces an output of $y = 5$, the other universities all produce $y = 1$, where y is some composite index of educational output. The inputs, x_1 and x_2, represent academic staff and students, respectively. The figure takes the form of a pyramid with its pinnacle at M. M is clearly an efficient university. However, so long as variable returns to scale are assumed, so too are universities A and B.[10]

Figure 3 near here

Under Cooper's approach, universities C and D would be deemed to be congested. Both are located on upward-sloping isoquant segments; this arises because $MP_1 > 0$ and $MP_2 < 0$ along segment BC, whereas $MP_1 < 0$ and $MP_2 > 0$ along segment AD. Both universities have $C_C = 0.2$, calculated as $\frac{1}{2}\{(0/6) + (4/10)\}$ for C and $\frac{1}{2}\{(4/10) + (0/6)\}$ for D. The evaluation is relative to university M in both cases.

University E is an interesting case because it is located on a downward-sloping isoquant segment; this arises because $MP_1 < 0$ *and* $MP_2 < 0$. Here $C_C = \frac{1}{2}\{(2/8) + (2/8)\} = 0.25$. The evaluation is again relative to university M. As in the case of C and D, E is deemed to be congested because a reduction in inputs is associated with a rise in output.

However, under Färe's approach, none of these three universities would be held to be congested! Instead, their inefficiency would be ascribed to the pure technical category. This finding can be explained by the fact that the projections onto the efficiency frontier occur along segment BA, at points C´, E´ and D´. In the identity $TE \equiv PTE \times SE \times CE$, where TE is overall technical efficiency, PTE is pure technical efficiency, SE is scale efficiency and CE is congestion efficiency, TE = 0.2, PTE = 0.4375, SE = 0.4571 and CE = 1 for all three universities.[11]

It is worth noting the circumstances in which a university *would* be found to be congested under Färe's approach. For instance, university C would need to be repositioned at a point such as C*, so that the ray OC* intersected the vertical line emanating from point B. Likewise, D would need to be repositioned at a point such as D*, so that the ray OD* intersected the horizontal line emanating from point A.[12] This exercise illustrates the point that an upward-sloping isoquant (negative marginal product for *one* of the factors) is necessary but not sufficient for congestion to occur under Färe's approach. In fact, for congestion to be identified, the relevant isoquant segment would need to be relatively steep or relatively flat.

University E is a rather different case: as Färe and Grosskopf (2000a, p. 32) themselves point out, a segment like CD on the unit isoquant would be ruled out of order by their axiom of *weak disposability*. In their world, isoquants may not join up in this 'circular' fashion. Weak disposability means that a proportionate rise in both x_1 and x_2 cannot reduce output. This eliminates the possibility that both factors might have negative marginal products, which is a necessary condition for a downward-sloping segment such as CD to occur.

What might congestion mean in the case of E? Cooper *et al.* (2001a, b) do not consider this issue, even though they criticize Färe's approach on the grounds of its alleged adherence to the law of variable proportions. The region CDM is defined in terms of the equation $y = 17 - x_1 - x_2$, which entails that *both* marginal products must be negative. For this to make economic sense in terms of the law of variable proportions, there would need to be some latent factor that was being held constant. Alternatively, but less plausibly, one might argue that diseconomies of scale had become so severe that equiproportionate increases in both factors were causing output to fall. Cherchye *et al.* (2001, p. 77) note that this second possibility would contravene Färe's axiom of weak disposability.

From this discussion, it is clear that we should not expect the competing approaches of Cooper and Färe to yield the same outcomes in terms of congestion.[13]

7. CONGESTION AND DISECONOMIES OF SCALE

Tone and Sahoo (2004) have proposed a new unified approach to measuring congestion and scale economies. This has several attractive features. The first is that, unlike Färe's method, negative marginal productivity always signals congestion.[14] Secondly, the analysis can easily be done using the *DEA-Solver Pro* software (www.saitech-inc.com). Thirdly, the output is comprehensive and easily understood. For simplicity, this procedure is referred to hereafter as Tone's approach.

Tone uses an output orientation. In fact, his approach is similar to Cooper's output-oriented method inasmuch as a BCC output-oriented model is used in the first stage. However, it differs in the second stage in its use of a slacks-based measure. To explain this approach, let us return to the example in Figure 3.

Like Cooper, Tone would find A, B and M to be BCC efficient and hence not congested. The remaining DMUs would have a congestion score of $\psi = 5$, reflecting the fact that M is producing five times as much output as any of them. *DEA-Solver* also provides us with a helpful figure for the *scale diseconomy*, ρ, for each congested university. For example, in the case of C, this is calculated as:

$$\rho = \frac{\%\ \text{change in y}}{\%\ \text{change in x}_1} = \frac{+400\%}{-40\%} = -10 \qquad (6)$$

Using the same method, we also get $\rho = -10$ for D. In the case of E, inputs fall by 25% on average, so that $\rho = -16$. These results suggest that congestion is equally serious for C and D but more serious for E. This finding is consistent with the outcome from Cooper's approach, where $C_C = 0.25$ for E but 0.2 for C and D. In Tone's terminology, we would describe E as being *strongly* congested (because both inputs are congested) but C and D as being *weakly* congested (because only one input is congested).

Having examined the different approaches to measuring congestion, we can now consider the outputs and inputs to be used in the DEA.

8. OUTPUT VARIABLES

It seems reasonable to argue that a university's output should be defined primarily according to the services it provides in terms of teaching, research, consultancy and other educational services. These aspects of a university's activities are captured here via the following variables:

- income from research grants and contracts in £ thousands;
- the number of undergraduate degrees awarded, adjusted for quality;
- the number of postgraduate degrees, diplomas and certificates awarded.

Sources of data and other details are given in Appendix A.

Income from Research Grants and Contracts

Research is clearly an important aspect of output in its own right. It may also indirectly influence the quality of teaching output by changing the focus of a university's academic staff.

Since universities sell their services to government and industry, the income received can be used to estimate the value of the output produced. However, the use of research income as a measure of output is problematic, since such income may be held to be an input into the research process rather than an output. Research income may also be distorted by differences in research costs across academic disciplines. On the other hand, research income is likely to reflect the perceived quality, as well as quantity, of research output and it should provide a more up-to-date picture of such output than, for example, the scores in some previous research assessment exercise (cf. Stevens, 2005, p. 357). Moreover, the necessary information is readily available. Indeed, in a study of this nature, one has little option but to use research income as a proxy for research output since annual data for most alternative variables are unavailable.

Many authors have, in fact, used research income as a proxy for research output; notable examples include Johnes (1997), Izadi *et al.* (2002) and Stevens (2005). For instance, Izadi *et al.* (2002, p. 66) argue that research grants may be regarded ‘as a measure of the market value of the research being undertaken [and that] their award characteristically reflects the grantee’s strong research performance in the recent past.’ One problem with this argument is that much research is speculative in nature; such research does not have an immediate market value, although it may yield valuable ‘spin offs’ at a later stage. Furthermore, one might argue that an important function of a university is to carry out research of *uncertain* market value.

Income from research grants and contracts includes, *inter alia*, income received from research councils, charities, central government, local authorities, health authorities, industry, commerce and public corporations. The variable includes income from both UK and overseas sources, although income from ‘other services rendered’ was excluded because of concerns about the comparability of some of the data.

Undergraduate Degrees

The total number of undergraduate degrees awarded is clearly an important measure of the output of any university. However, an obvious shortcoming of this measure is that it fails to take any account of the *quality* of the degrees awarded.

One way of taking quality into account would be to multiply the number of degrees awarded by the proportion of students gaining ‘good’ degrees, defined in some way. This proportion should be positively related to the quality of teaching. With degree results, there is a choice between a very narrow definition of quality – the proportion of first-class honours degrees awarded to undergraduate students – and a broader definition comprising both firsts and upper seconds (cf. Stevens, 2005, p. 356). Here we have opted to use a broader definition of quality, so that the output variable becomes the *number* of firsts and upper seconds awarded.

Nonetheless, some problems with this output variable must be recognized. The first is that, by focusing on firsts and upper seconds, the resources employed in teaching the other graduates are being ignored. Secondly, students' achievements depend not only on the quality of teaching but also on their effort, ability and initial qualifications. Unfortunately, it was not possible to adjust for any of these attributes. A third potential problem is the possible variation, both across institutions and intertemporally, in the implicit standards set for particular classes of degree and also in the assessment schemes used (e.g. the weighting given to coursework).

We shall be addressing the first potential problem noted above by carrying out a sensitivity analysis using *all* undergraduate awards as the output variable. As regards the third potential problem, it is worth noting that the steady long-term rise in the proportion of firsts and upper seconds awarded would not be a matter for concern, so long as it represented a genuine enhancement in the quality of graduates.[15] However, even if it did not, a common trend across all universities would not affect the DEA results.

A final caveat is worth noting: the only output recognized is degrees awarded to final-year students, despite the fact that all undergraduates are used as an input. Nonetheless, this should not present a serious problem unless the number of students was growing at substantially different rates in different universities.

Postgraduate Qualifications

For simplicity, and in order to avoid artificially boosting the efficiency scores, masters degrees and doctorates, along with postgraduate certificates and diplomas, were aggregated into a single variable.[16] A disadvantage of this is, of course, that variations across universities in the proportion of each type of postgraduate qualification are thereby ignored. This variable also fails to take account of possible differences in the quality of postgraduate qualifications.

9. INPUT VARIABLES

The following input variables are used in the DEA analysis:

- the number of full-time equivalent undergraduate students (X_1);
- the number of full-time equivalent postgraduate students (X_2);
- academic staff expenditure in £ thousands (X_3);
- other expenditure in £ thousands (X_4).

See Appendix A for sources of data and other details. Some comments on inputs X_3 and X_4 are made below.

Academic Staff Expenditure

Input X_3 measures a university's total expenditure on academic staff. As such, it has the merit of being measured in the same units as input X_4. A possible demerit of X_3 is that staffing expenditure will vary with the proportion of staff on different grades and only approximately with the number of staff hours available for teaching, research, administration, etc. Therefore, an alternative variable – the full-time equivalent number of academic staff – is considered in a sensitivity analysis.

Other Expenditure

This variable measures a university's total expenditure *minus* its academic staff expenditure. It comprises expenditure on academic cost centres, academic services, administration and central services, premises, residences and catering, and on research grants and contracts.

10. TECHNICAL EFFICIENCY

Before considering the issue of congestion, it is worth examining the overall *technical efficiency* (TE) of the former polytechnics in the period 1995/6 to 2003/4. Table 1 exhibits the findings from three alternative models.[17] *Model 1* is the one outlined above, in which the output of undergraduate qualifications is measured by the number of firsts and upper seconds awarded. In *Model 2*, this output variable is replaced by *all* undergraduate awards.[18] Finally, *Model 3* is a modified version of *Model 2*, whereby expenditure on academic staff is replaced by the number of full-time equivalent staff. It should be noted that the sample comprises 41 institutions up to 2001/2 but 40 thereafter. This is due to the merger of London Guildhall University and the University of North London to form London Metropolitan University.[19]

Table 1 near here

Along with the annual unweighted arithmetic mean (UAM) TE scores for each model, Table 1 also shows the corresponding weighted arithmetic mean (WAM) scores, which were calculated using the number of students in each university as a weight. This was done to take account of the unequal size of universities (see Appendix B). The unweighted results, which are also illustrated in Figure 4, will be examined first.

Figure 4 near here

If we ignore the erratic results for the first three years, then the unweighted mean TE scores from Model 2 exceed those from Model 1 in five years out of six. This is evident from both Table 1 and Figure 4. This outcome probably reflects the fact that it is possible, with Model 2, to substitute one type of undergraduate award for another, while keeping the overall number of awards constant, e.g. an upper second could be replaced by a lower second. This would tend to moderate the intertemporal fluctuations in output and lessen the variation in efficiency across universities. This, in turn, would tend to raise the mean TE scores.

If we again ignore the first three years, then Table 1 and Figure 4 also reveal that the unweighted mean TE scores from Models 2 and 3 are not that different. What is more, there is no tendency for these results to diverge in a systematic way. This suggests that it may not make much difference to the conclusions whether one measures the input of academic staff in terms of full-time equivalents or expenditure. The close relationship between Models 2 and 3 for the last six years was confirmed by the finding of a strong positive correlation of 0.943

between the 244 individual TE scores generated by each model. By contrast, r = 0.739 for Models 1 and 2.

If we now look at the first three years, it is surprising that the minima of the graphs for Models 2 and 3 occur in different years. However, this may merely reflect possible errors in the data for full-time equivalent academic staff. This series is much more erratic than the corresponding one for academic staff expenditure, and we observed some very large annual changes in the FTE figures for some institutions, especially in the earlier years.

With regard to weighting, the ‘Difference’ column in Table 1 shows that this procedure enhances the mean scores for Model 1, albeit by a modest amount in most cases. For Models 2 and 3, the weighting slightly raises the mean scores in all years apart from 2003/4. Taking the results as a whole, however, there is a clear tendency for the scores from the different models to converge during the period under review.

Whilst the mean levels of technical efficiency are generally fairly high, there is no evidence of an upward trend, especially from 1998/9 onwards. Indeed, all of the models show that the rise in 2002/3 was offset by a downturn in the final year. This is true for both weighted and unweighted scores. It is worth noting too that all models record a fall in the number of frontier universities in the final year. However, it should be borne in mind that the TE scores do not measure technical efficiency in an absolute sense but instead measure it relative to the frontier in each year. Hence the drop in the mean TE scores in 2003/4 could mean that the universities were moving further away from a static frontier or, alternatively, that the frontier had shifted outwards.[20] It may be noted, finally, that the mean TE scores being discussed here are somewhat lower than the comparable scores we obtained for 45 older British universities over the same period (Flegg and Allen, 2007). This suggests a greater degree of heterogeneity in the sample of former polytechnics.

11. CONGESTION: COOPER'S PROCEDURE

For Cooper's procedure, the first step was to work out C_C, the average proportion of congestion in the inputs used by each university in each year. These scores were then averaged, first over all universities, and then over the congested universities alone, to get the respective values for $\overline{C}_C$. Both weighted and unweighted means were computed for the whole sample. The results are displayed in Table 2. For simplicity, the discussion is confined to Model 2.

Table 2 & Figure 5 near here

Table 2 shows that the differences between the weighted and unweighted means are mostly relatively small. Therefore, again for simplicity, only the latter will be discussed here. These unweighted means indicate that congestion for the whole sample fell from an average of 5% of inputs in 1995/6 to a more modest 3.25% in 2003/4. This tendency for the value of $\overline{C}_C$ to fall over the period as a whole is also apparent from Figure 5.

However, in assessing the degree of congestion, it may be more appropriate to focus on the congested universities alone. For instance, in 2003/4, $\overline{C}_C = 0.0564$ for the 23 congested universities, compared with 0.0325 for the whole sample. Looked at in this way, with congestion averaging 5.64% of inputs, the problem appears more serious. The impact of focusing on the congested universities is also clearly demonstrated in Figure 5.

Figure 5 reveals that the values of $\overline{C}_C$ are less stable from 1998/9 onwards. Here it is interesting to see that Cooper's measure of congestion first falls and then rises in the final two years, whereas mean technical efficiency does the opposite.

More light can be shed on the extent of the problem by examining the individual values of C_C for 2003/4. These scores, which are presented in Appendix B, range from 0.009 (Brighton) to 0.137 (Glasgow Caledonian). 13 of the 23 congested universities have $C_C > 0.040$. What we now need to do is to see how robust these findings from Cooper's approach are, by considering the results from alternative approaches. For simplicity, the discussion will again be confined to Model 2 and to the unweighted results.

12. CONGESTION: TONE AND COOPER

With Tone's procedure, the following transformation was used: $C_T \equiv 1 - 1/\psi$, where $\psi \geq 1$ is the congestion score generated by *DEA-Solver Pro*. C_T can thus be compared directly with Cooper's congestion score, C_C, as both have a range from 0 (no congestion) to 1 (maximum congestion). The annual unweighted arithmetic mean values of C_T and C_C are displayed in Table 3 and illustrated in Figure 6.

Table 3 & Figure 6 near here

The results for the whole sample show that Tone's procedure indicates the most congestion. Indeed, in six years out of nine, $\overline{C}_T$ is clearly above $\overline{C}_C$. This is an interesting outcome because the two approaches generate exactly the same set of 23 congested universities in 2003/4; where the procedures differ is in terms of the severity of the problem identified in each university (see Appendix B). Here it is worth noting that not only is Tone's method apt to find more congestion but it also almost always gives a different ranking of the congested universities.

For the period as a whole, we found only three instances out of 367 where Tone and Cooper would disagree about whether a particular university was or was not congested (all of these had $C_T = 1$ but $C_C < 1$). This close matching of the universities held to be congested by the two approaches can be attributed to the fact that both use an output-oriented version of the BCC model as their starting point. Thus scale effects are removed prior to attempting to measure congestion. Also, only those universities deemed to be inefficient in terms of the BCC model are examined for possible congestion. Therefore, even though Cooper and Tone measure congestion somewhat differently, they are still looking at the same set of potentially congested universities.

The fact that almost all congested universities have different values of C_T and C_C can be explained by the different way in which congestion is measured. For Cooper, an input exhibits congestion if it has a non-zero BCC slack, while the amount of congestion is held to be equal to that slack.[21] The average proportion of congestion over all inputs is then calculated. By contrast, Tone's procedure measures the potential increase in output from eliminating the congestion of inputs.[22] Given this difference in approach, it would be most surprising if the results *did* end up being very similar. Indeed, in our study of 45 older British universities over the same period (Flegg and Allen, 2007), we found that $\overline{C}_C$ typically *exceeded* $\overline{C}_T$, the opposite of the result obtained here!

It is interesting that Tone's procedure indicates a *rise* in congestion over the period as a whole, from 4.9% in 1995/6 to 5.8% in 2003/4, whereas Cooper's method indicates a fall from 5% to 3.25%. If we now focus on the 23 congested universities in 2003/4, Cooper's method indicates congestion of 5.6%, whereas Tone's procedure yields a much higher figure of 10.1%.

Tone's measure also tends to track the path of technical efficiency more closely. This is shown by the correlation between the 367 individual TE scores and each measure: -0.699 for C_T but -0.379 for C_C. It is noticeable too how the rise and then fall in $\overline{C}_T$ in the first two years coincides with opposite movements in mean technical efficiency (see the graph for *Model 2* in Figure 4). These findings suggest that the fluctuations in TE scores may, to some extent, be due to underlying changes in congestion.

A helpful attribute of Tone's approach is the information it provides, via a parameter ρ, about diseconomies of scale. Table 3 shows the annual arithmetic mean values of ρ for the congested universities. Consider two examples: given a 1% decrease in congested inputs, the results indicate a potential rise in output of 8.5% on average in 1998/9 but only 4.6% in 2003/4. This suggests that congestion was more serious in 1998/9. However, given its sensitivity to extreme values, $\overline{\rho}$ is not a very reliable measure of the amount of congestion in a given year and it is more useful to examine the values of ρ for individual universities.

To illustrate, let us consider the results for 2003/4. Appendix B shows that the value of ρ in that year ranged from −11 for London South Bank to −0.2 for Portsmouth. These figures suggest that a 1% reduction in congested inputs could potentially have increased output by 11% in London South Bank but by only 0.2% in Portsmouth. It should be noted, however, that only congested inputs are considered in the calculation of ρ. Likewise, only those outputs affected by congestion are taken into account, i.e. those where the results indicate a potential rise in output. Hence ρ does not measure the ratio of the overall percentage changes in inputs and outputs.

13. CONGESTION: FÄRE'S APPROACH

When measuring congestion, Cooper and Tone both employ an output-oriented approach, with variable returns to scale (VRS) as the underlying technology. It is, therefore, worth examining how sensitive the results are to a change in the assumed technology. Färe's approach offers a convenient way of doing this.

In their earlier work, Färe and Grosskopf assumed an absence of congestion when measuring scale effects, and only then allowed for the possibility of congestion.[23] This meant that, like Cooper and Tone, they were assuming VRS initially. However, Färe and Grosskopf (2000b) have highlighted the problems associated with distinguishing between scale inefficiency and congestion; they point out that the congestion score will depend on the *order* in which technical efficiency (TE) is decomposed. Therefore, where congestion is anticipated on *a priori* grounds, Färe and Grosskopf recommend that, rather than assuming VRS technology, one should base one's measurements on constant returns to scale (CRS). This issue will be explored here by using an input-oriented version of their approach.

To clarify why the order of decomposition matters, consider the identity:

$$\mathrm{TE} \equiv \mathrm{PTE} \times \mathrm{SE} \times \mathrm{CE} \qquad (7)$$

where PTE is pure technical efficiency, SE is scale efficiency and CE is congestion efficiency. Crucially, in this identity, TE and the product SE × CE are unaffected by the order of the decomposition but the individual values of SE and CE *are* affected.

Figure 7 & Table 4 near here

A glance at Figure 7 is all that is required to see that we get appreciably more 'congestion' if we assume CRS rather than VRS.[24] This is demonstrably true for all years apart from 2000/1. What is more, the gap between the $\overline{\mathrm{C}}_{\mathrm{F,CRS}}$ and $\overline{\mathrm{C}}_{\mathrm{F,VRS}}$ graphs shows no sign of disappearing. It is also interesting that, of the three measures, Cooper's measure, $\overline{\mathrm{C}}_{\mathrm{C}}$, clearly indicates the least congestion, although 2000/1 is once again an exception. These findings are substantiated in Table 4.

Figure 8 near here

Figure 8 illustrates the relationship between Färe's VRS-based measure and that of Tone. The detailed results are presented in Table 4. One can see that Färe's measure typically exceeds that of Tone and that there are only two years where the converse is true. However, on average, the differences are fairly small, as shown by a mean difference of only 0.0035.

Table 5 near here

To shed some more light on the relationships among the different measures, correlation coefficients were calculated using the raw congestion scores (n = 367). Table 5 shows the results. As expected, Färe's VRS-based measure is very strongly correlated with that of Tone. The fact that this correlation is 0.904 rather than unity can be attributed to the different orientation and to the different ways in which congestion is measured.

Färe's CRS-based measure is also very strongly correlated with that of Tone. This result was not expected but it reflects the fact that Färe's two measures are themselves strongly correlated (r = 0.855). As expected, Table 5 shows that Cooper's measure is not strongly correlated with any of the other three measures.

The correlation analysis shows that the four measures are positively associated, yet the strength of this correlation varies substantially and some measures appear to be more substitutable than others. Even so, the correlations need to be interpreted with care. For instance, $C_{F,VRS}$ is a much closer substitute for C_T than $C_{F,CRS}$, even though both have a correlation of $r \approx 0.9$ with C_T. This is because $C_{F,CRS}$ is likely to overestimate C_T. More detailed information is given in Appendix B, where the individual results for 2003/4 are tabulated.

It is worth emphasizing, finally, that the different measures do not indicate similar trends in congestion over the period as a whole: whereas Färe's two measures suggest little change in congestion, Tone's measure points to a modest rise and Cooper's measure indicates a clear but rather bumpy downward trend!

14. SCALE INEFFICIENCY AND CONGESTION

Appendix B shows a set of TE (technical efficiency) and SE (scale efficiency) scores for individual universities in 2003/4. The SE scores were calculated by taking the ratio of the efficiency scores from the CCR and BCC models. This appendix also shows the scores from the four alternative measures of congestion.

The individual results reveal a diversity that is hidden when looking at annual means. A good example is Thames Valley, which has the lowest TE score in the sample. This score suggests that Thames Valley was producing only 51% of its potential output in 2003/4. As to the causes of this inefficiency, Färe and Tone would regard Thames Valley as being chronically congested, whereas Cooper would find only a moderate amount of congestion. This issue is taken up later in the paper. Also of interest is Thames Valley's SE score of

0.9729, which indicates that it was operating at a high level of scale efficiency, with only 2.7% of potential output being lost as a result of its failure to achieve full scale efficiency.

Another example worth considering is Manchester Metropolitan. This university has TE = SE = 0.8547. The fact that its TE and SE scores are identical indicates that it was operating on the BCC frontier ($\phi^* = 1$). According to both Cooper and Tone, *all* of its inefficiency would be attributed to its inappropriate scale, so that any congestion would be ruled out. However, under the CRS-based version of Färe's approach, congestion is possible and, indeed, Manchester Metropolitan has $C_{F,CRS} = 0.0386$. There are five other universities in a similar situation.

15. DECOMPOSING CONGESTION

An advantage of Cooper's approach is that it is possible to measure, for each congested university, the contribution of each input to the observed amount of congestion. Table 6 takes a closer look at this feature of his approach, using annual means to summarize the data. The table shows a decomposition by input of the annual unweighted mean value of C_C.

Table 6 near here

The results for Model 1 indicate that an overabundance of undergraduate students was the largest single cause of congestion in the former polytechnics during the period under review. On average, such students accounted for 34.5% of the value of $\overline{C}_C$. However, the results suggest that academic overstaffing was also a major cause of congestion in these new universities! Indeed, at 30.8%, the average share of academic staff is not far behind that of undergraduates. By contrast, the results suggest that postgraduates and 'other expenditure' played a noticeably smaller role in generating congestion.

The pre-eminence of undergraduates in generating congestion is confirmed by the results from Model 2. Indeed, there is now a noticeably wider gap between the average shares of undergraduates and academic staff. With an average share of 26.0%, academic staff are now clearly in second place. What is surprising is that the switch from a narrower to a broader measure of undergraduate output has had so little impact on the share of undergraduates. We did not expect academic staff to be the main beneficiaries of the change in model. As regards postgraduates and 'other expenditure', the results show that these two inputs have gained in importance, although their respective shares are still of comparable size. It is worth noting that these various changes in shares have little impact on the overall mean value of $\overline{C}_c$.

As expected, the results for Models 2 and 3 are broadly similar and there is again hardly any change in the overall mean value of $\overline{C}_c$. Undergraduates are shown once more to be the largest single factor underlying congestion, with an average share that is only slightly lower than before. Nonetheless, some changes are worth noting. In particular, as a result of using full-time equivalents rather than expenditure, there is a further appreciable fall in the average share of academic staff and concomitant rise in the shares of postgraduates and 'other expenditure'. The average shares of these three inputs are now of roughly comparable size.

Whilst the role attributed to academic staff in generating congestion is not a dominant one, it is still puzzling. What the findings suggest is that, other things being equal, a reduction in academic staffing could have *raised* the output of congested universities in terms of earnings from research and consultancy, as well as undergraduate and postgraduate qualifications obtained. One possible explanation is that overstaffing caused congestion of facilities such as libraries, office accommodation, etc. and this, in turn, caused a fall in output. This could be relevant if the frontier universities were generally better endowed than the congested universities. It is also possible that the presence of 'surplus' staff in the congested universities might be indicative of institutional inefficiency in a broader sense.

The role attributed to 'other expenditure' in all three models is equally puzzling. What this suggests is that, beyond a certain point, extra expenditure actually reduced congested universities' output. However, a possible explanation is in terms of the *mix* of expenditure.

‘Other expenditure’ is a very broadly defined input variable, comprising expenditure on academic cost centres, academic services, administration and central services, premises, residences and catering, and on research grants and contracts. It is conceivable that a rise in the proportion of other expenditure devoted to research could impact adversely on the output of undergraduate and postgraduate qualifications, although it might stimulate research activity. Conversely, a fall in this proportion could have the opposite effect. Another possible explanation is in terms of excessive spending on administration, which might reduce a university’s efficiency and hence output in terms of research and qualifications awarded.

16. DISAGGREGATED RESULTS

Some additional insights can be gleaned from the results for individual universities, which are exhibited in Table 7. These results are based on Model 2 and relate to 2003/4.

Table 7 near here

In terms of Cooper’s measure, Glasgow Caledonian and Sunderland are clearly the most congested universities, with Teesside not far behind. However, the underlying causes are rather different in each case. For instance, whereas Glasgow Caledonian has an overabundance of academic staff, Sunderland has excessive ‘other expenditure’. In addition, both have too many undergraduates. In the case of Teesside, the salient factors are academic overstaffing and too many undergraduates.

Of the four factors underlying congestion, an excessive number of undergraduates is undeniably the pre-eminent one, affecting all but four congested universities. This problem is especially serious in Teesside, Central England, Nottingham Trent, Anglia and Glasgow Caledonian. By contrast, academic overstaffing, whilst still a cause for concern, is both less prevalent and less acute in most cases.

Earlier in the paper, it was noted that Thames Valley was only moderately congested in terms of Cooper’s measure. From Table 7, one can see that its C_C score is below average for the congested universities; this occurs because its congestion in terms of undergraduates, which is above average, is outweighed by negligible congestion elsewhere.

It is interesting that only five of the new universities are congested in terms of postgraduates. Here East London and, to a lesser extent, London Metropolitan are conspicuous in terms of having too many postgraduates. 'Other expenditure' is a determinant of congestion in nine of the new universities. Sunderland and, to a lesser extent, Northumbria stand out as having particular problems in this respect.

Several cases of more moderate congestion are also shown in Table 7. For example, both London South Bank and West of England have below-average congestion. In both cases, the congestion can be attributed to academic overstaffing and, to a lesser extent, to having too many undergraduates.

The findings discussed above offer an interesting contrast with the results we obtained for 45 older British universities over the same period (Flegg and Allen, 2007); these gave a less prominent role to academic overstaffing, and a more prominent role to postgraduates, although the overall incidence of congestion was noticeably less.

17. CONCLUSION

This paper has used data envelopment analysis (DEA) to examine the performance of 41 former British polytechnics that became universities in 1992, using annual data for the period 1995/6 to 2003/4. These new universities differ from the older universities in many ways, especially in terms of their far higher student : staff ratios and substantially lower research funding per member of staff. What is more, this under-resourcing increased during the period under review, as exemplified by a further rise in the student : staff ratio from 17.5 to 19.3.[25]

The issue that has been explored here is whether this under-resourcing of the new universities has caused them to be 'congested', in the sense that their output – as measured by the number of undergraduate and postgraduate awards, along with earnings from research and consultancy – has been lower than it might otherwise have been.

Three alternative approaches to measuring congestion were examined: the conventional approach of Färe and Grosskopf, the alternative proposed by Cooper *et al.*, and a new method developed by Tone and Sahoo. In addition, in the case of Färe and Grosskopf's approach, two versions were considered: one assumed constant returns to scale (CRS), while the other assumed variable returns to scale (VRS). To check the sensitivity of the results to different specifications, three alternative DEA models were formulated.

The different measures of congestion produced rather different results in terms of the degree of congestion indicated. Tone and Sahoo's method and the VRS-based version of Färe and Grosskopf's approach were the most similar of the four methods. For instance, in 2003/4, the former method indicated congestion of 5.8%, on average, across the 40 universities, whereas the latter method indicated 6.0%. When the results were averaged over the 23 congested universities, the figures were still similar, albeit much higher, viz 10.1% and 10.4%, respectively. Cooper's method generated the lowest average congestion scores of the four methods: 3.25% for the whole sample and 5.6% for the congested universities.

Switching from VRS to CRS had a marked impact on the results generated by Färe and Grosskopf's approach: the mean congestion scores were substantially higher in almost all years. What is more, this method consistently produced the highest congestion scores of all the methods examined here. For instance, the mean score for the whole sample was 7.0% in 2003/4, well above the 6.0% for the VRS-based variant of their procedure, the 5.8% for Tone and Sahoo's method and the 3.25% for Cooper's method.

It is worth noting too that the different measures did not indicate similar trends in congestion over the period as a whole: whereas Färe's two measures suggested little change in congestion, Tone's measure pointed to a modest rise and Cooper's measure indicated a clear but rather bumpy downward trend!

The underlying causes of congestion were explored via a decomposition analysis based on Cooper's procedure. This revealed that an overabundance of undergraduate students was the largest single cause of congestion in the former polytechnics during the period under review. On average, based on our Model 2, such students accounted for 42.3% of the value of Cooper's congestion score in 2003/4. Less plausibly, the results suggested that academic overstaffing was also a major cause of congestion in the new universities! Here the results indicated a share of 29.5%. By contrast, the results suggested that postgraduates (12.3%) and 'other expenditure' (16.0%) played a noticeably smaller role in generating congestion.

To put these findings into context, it may be noted that the figure of 42.3% for congestion due to undergraduates is equivalent to 751 'surplus' undergraduates, on average, for all universities, or an average of 1306 for the 23 congested universities. The comparable figures for postgraduates are 53 and 91.5, respectively. In the case of academic staff, the mean expenditure on 'surplus' staff was £1,437,000 for all universities or £2,499,000 for the congested universities alone. Finally, for 'other expenditure', the relevant figures are £1,495,000 and £2,600,000, respectively.

How realistic are the above findings likely to be? On the one hand, one might argue, as some have done, that Cooper's approach is deficient. Certainly, in this context, one could question the realism of the sizable role attributed to academic staff and to 'other expenditure' in generating congestion. On the other hand, Cooper's method generated the lowest average congestion scores of the four methods, so that the above figures may well represent minima rather than maxima. It is also worth noting that the findings were not greatly affected by a change in the DEA model employed.

In terms of implementing the findings of this study, one important caveat needs to be stated: it may well be much easier to comprehend the causes of congestion than to realize the potential gains in output from eliminating such congestion.

With respect to the different results generated by the alternative methods of measuring congestion, one should not lose sight of the fact that the three VRS-based methods almost invariably identified the same universities as being congested. Where they differed was in terms of the severity of congestion in the universities affected. Since the different methods all have their respective merits and demerits, yet produce different results, it would seem sensible not to rely on a single method. For the same reason, relying upon the rankings generated by a single method would be unwise.

As regards the generality of the results obtained here, it is clear that one is likely to find more 'congestion' with a CRS-based model than with a VRS-based model. Another general finding is that the VRS-based variant of Färe and Grosskopf's approach should generate broadly similar results to Tone and Sahoo's method. However, with respect to Cooper's method *vis-à-vis* the VRS-based variant of Färe and Grosskopf's approach, there does not appear to be any general relationship. This comment is based on the computational differences between the two methods, along with the fact that the findings obtained here conflict with those of Flegg and Allen (2007) for the older British universities. It does seem probable that different samples will produce different results.[26]

From the results presented in this paper, it seems fair to conclude that many of the former polytechnics are congested to a considerable degree. This conclusion is bolstered by the fact that the findings were not greatly affected by changes in the DEA models employed and also by the fact that the alternatives to Cooper's measure of congestion invariably indicated more rather than less congestion. It is also worth noting that the former polytechnics appear to be more affected by congestion than do the older universities (see Flegg and Allen, 2007).

There are clearly some areas where this study could be built upon. The first is that a Malmquist analysis could be employed to distinguish between fluctuations in congestion brought about by shifts in the efficiency frontier, as opposed to movements towards or away from this frontier. Secondly, use could be made of the facility in *OnFront*, whereby one can restrict consideration to a subset of inputs most likely to be affected by congestion. Finally, it would be interesting to explore what effect changing the definitions of some of the inputs and outputs would have on the findings (e.g. allocating points to different classes of undergraduate degree, as in Johnes, 2006).

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We would particularly like to thank Timo Kuosmanen for some very helpful suggestions. We would also like to thank Kate Lang of the Higher Education Statistics Agency (HESA) for the efficient way in which she answered our many queries and produced the data we needed. See Appendix A for details.

ENDNOTES

1. The basic data used in this study were obtained, either directly or indirectly, from the Higher Education Statistics Agency (HESA). See Appendix A for details.
2. In 2003/4, for example, 13.2% of undergraduates in the 45 older universities gained first-class degrees and 49.3% gained upper seconds, whereas the proportions in the ex-polytechnics were 7.7% and 39.0%, respectively. *Source*: Authors' own calculations using HESA data.
3. The number of full-time equivalent students in the ex-polytechnics rose by 15.1% between 1995/6 and 2003/4, compared with a rise of 26.4% in the 45 older universities.
4. Congestion in the older universities is examined in Flegg and Allen (2007).
5. The student : staff ratio in the ex-polytechnics rose from 17.5 in 1995/6 to 19.3 in 2003/4.
6. Figure 2 is adapted from Tone and Sahoo (2004, Figure 2).
7. For a detailed discussion of the properties of the CCR and BCC models, see Cooper *et al.* (2000a).
8. For details of this software, see www.emq.com.
9. A diagram similar to Figure 3 is the subject of a debate between Cherchye *et al.* (2001) and Cooper *et al.* (2001a, b).
10. A and B would be inefficient under constant returns to scale whereas M would be efficient.
11. This was confirmed using *OnFront* and an input-oriented model.
12. CE = Oc/OC* and CE = Od/OD* for the repositioned C and D, where CE = 0.8 in both cases.
13. For a more detailed discussion, see Flegg and Allen (2006).
14. We are indebted to Kaoru Tone for confirming this point.
15. In 2003/4, for example, 7.7% of undergraduates in the ex-polytechnics gained first-class degrees and 39.0% gained upper seconds, compared with 4.3% and 37.8%, respectively, in 1995/6. *Source*: Authors' own calculations using HESA data.
16. DEA efficiency scores tend to rise as the number of variables increases, thereby reducing the discriminatory power of the technique.
17. These TE scores were obtained (using *DEA-Solver-Pro*) from the CCR model, which assumes constant returns to scale (CRS) and no congestion. The orientation of the model has no effect on the TE scores under CRS. *OnFront* generated identical results.
18. This broader variable encompasses all undergraduate degrees, as well as 'other undergraduate awards' such as certificates and diplomas in business, computing, engineering, medicine, nursing and technology, along with higher national diplomas, certificates and diplomas of higher

education, etc. For the ex-polytechnics, these ‘other awards’ gained in importance, rising from 27.7% of all undergraduate awards in 1995/6 to 34.3% in 2003/4. In some cases, these other awards are a default qualification rather than one that would be sought in its own right.

19. Although DEA does not require a balanced panel of DMUs, we nonetheless experimented with models in which the data for London Guildhall and North London were pooled to form a single entity in the first seven years. However, it made little difference whether these two universities were combined into a single DMU or analysed separately. The explanation for this is that neither university appeared on the frontier in any year.
20. To discriminate between these two possibilities would require a Malmquist analysis (see Flegg *et al.*, 2004). However, an analysis of this kind is beyond the scope of the present paper.
21. This statement presupposes that there are no DMUs like E in Figure 2.
22. Tone uses an output-oriented slacks-based measure in his projection of the congested universities onto the BCC frontier. For an explanation of this SBM procedure, see Tone (2001).
23. See, for example, Byrnes *et al.* (1984), and Färe *et al.* (1985a).
24. The calculations were carried out using *OnFront*. For comparative purposes, congestion efficiency (CE) scores were converted into *in*efficiency scores by defining $C_F \equiv 1 - CE$.
25. In the older universities, the student : staff ratio rose from 7.5 in 1995/6 to 9.4 in 2003/4.
26. Cf. Färe and Grosskopf (2000a, p. 33), who suggest that their method would typically yield *less* congestion than Cooper’s method. Their reasoning here is that, with their method, only part of any input slack would be treated as representing congestion.

Appendix A. Sources and definitions

Most of the data used in this study were obtained directly from various issues of the following publications of the Higher Education Statistics Agency (HESA):

- *Resources of Higher Education Institutions* (RHEI)
- *Students in Higher Education Institutions* (SHEI)

See HESA (various years). In some cases, noted below, data were obtained directly from HESA under contract. We decided to omit 1994/5 from our study because of missing data for Luton and Robert Gordon universities. The results for 1995/6 should be treated cautiously because of possible problems with the data on full-time equivalent numbers of students and staff.

Some key information on the variables used in this study is given below. More detailed information is given in the HESA publications mentioned above.

- *Income from research grants and contracts*

Because of concerns about the comparability of some of the data, this variable excludes data on what HESA defines as income from ‘other services rendered’. Source: RHEI, Table 3 up to 2001/2, Table 1c thereafter.

- *Number of undergraduate and postgraduate qualifications awarded*

The qualifications data published in SHEI could not be used for two reasons:

(i) the severe rounding of the published data from 1999/2000 onwards;

(ii) the unspecified qualifications of ‘dormant students’ from 1995/6 to 1999/2000.

Fortunately, we were able to obtain the necessary data directly from HESA. For Luton in 1997/8, the figures for undergraduate degrees awarded were not separated into classes, so we used interpolation to estimate the missing figures for use in Model 1.

- *Full-time equivalent undergraduate and postgraduate students* (X_1 and X_2)

HESA did not publish full-time equivalent numbers for 1994/5 and 1995/6, owing to concerns about the quality of the data. Although we were able to obtain the unpublished data directly from HESA, we have used the figures for 1995/6 in our study with some reservations. Data from 1996/7 onwards were obtained from SHEI, Table 0b.

- *Academic staff expenditure* (X_3)

Source: RHEI, Table 7 up to 2001/2, Table 2b thereafter.

- *Other expenditure* (X_4)

Variable X_4 was calculated by subtracting what HESA defines as 'other expenditure' from each university's total expenditure and then deducting academic staff expenditure (X_3). HESA's 'other expenditure' was not included, as we were concerned about the comparability of some of the data.

Source: RHEI, Tables 6 and 7 up to 2001/2, Tables 2a and 2b thereafter.

- *Full-time equivalent number of academic staff*

The HESA data on this variable were downloaded from http://www.data-archive.ac.uk. It should be noted that we have some concerns about the reliability of the data for 1995/6. In particular, the aggregate student : staff ratio for that year looks unrealistically high.

Appendix B. Individual results for 2003/4: Model 2

						Färe				Tone			Cooper	
University	Weight	**TE**	RANK	**SE**	RANK	$C_{F,CRS}$	RANK	$C_{F,VRS}$	RANK	C_T	RANK	ρ	C_C	RANK
Abertay Dundee	0.007	1	1	1	1	0	1	0	1	0	1		0	1
Anglia Polytechnic	0.028	0.7886	35	0.9883	17	0.2114	37	0.2009	37	0.2020	37	-1.84	0.0624	32
Bournemouth	0.020	0.8520	28	0.9806	20	0.1214	34	0.1442	34	0.1312	34	-4.80	0.0320	25
Brighton	0.024	0.8305	31	0.9753	21	0.0035	13	0.0034	18	0.0141	22	-0.95	0.0088	18
Central England	0.029	0.9203	19	0.9573	26	0.0797	27	0.0439	27	0.0387	25	-3.36	0.0844	35
Central Lancashire	0.033	0.9510	16	0.9510	28	0.0490	22	0	1	0	1		0	1
Coventry	0.022	0.9700	14	0.9700	25	0.0300	18	0	1	0	1		0	1
De Montfort	0.030	1	1	1	1	0	1	0	1	0	1		0	1
Derby	0.018	0.9088	23	0.9506	29	0.0632	25	0.0310	23	0.0206	23	-8.19	0.0184	20
East London	0.019	0.8373	29	0.9939	16	0.1627	36	0.1348	32	0.1576	35	-2.41	0.0920	36
Glamorgan	0.022	0.7696	38	0.9987	15	0.1180	33	0.2162	38	0.2293	38	-4.03	0.0224	22
Glasgow Caledonian	0.022	0.9599	15	0.9991	13	0.0401	20	0.0365	25	0.0393	26	-3.73	0.1370	40
Greenwich	0.025	1	1	1	1	0	1	0	1	0	1		0	1
Hertfordshire	0.030	0.7736	37	0.9384	31	0.0774	26	0.0504	28	0.0462	28	-1.59	0.0389	27
Huddersfield	0.021	1	1	1	1	0	1	0	1	0	1		0	1
Kingston	0.027	0.8122	33	0.8943	39	0	1	0.0081	20	0.0043	19	-3.77	0.0100	19
Leeds Metropolitan	0.034	0.9089	22	0.9089	37	0.0192	16	0	1	0	1		0	1
Lincoln	0.017	1	1	1	1	0	1	0	1	0	1		0	1
Liverpool J. Moores	0.027	1	1	1	1	0	1	0	1	0	1		0	1
London Metro	0.036	0.8319	30	0.9023	38	0.0562	23	0.1680	35	0.0781	29	-5.78	0.0584	31
London South Bank	0.022	0.7353	39	0.9988	14	0.2647	39	0.2335	39	0.2639	39	-11.00	0.0474	29
Luton	0.013	1	1	1	1	0	1	0	1	0	1		0	1
Manchester Metro	0.045	0.8547	27	0.8547	40	0.0386	19	0	1	0	1		0	1
Middlesex	0.027	0.9720	13	0.9720	24	0.0280	17	0	1	0	1		0	1
Napier	0.015	0.8946	24	0.9738	22	0.1054	32	0.0851	29	0.0813	30	-10.12	0.0444	28
Northumbria	0.032	0.8637	25	0.9492	30	0.1053	31	0.0939	30	0.0901	32	-5.70	0.0766	34
Nottingham Trent	0.039	0.9917	12	0.9999	12	0.0083	14	0.0082	21	0.0083	20	-7.43	0.0961	37
Oxford Brookes	0.023	1	1	1	1	0	1	0	1	0	1		0	1
Paisley	0.013	1	1	1	1	0	1	0	1	0	1		0	1
Plymouth	0.034	0.8256	32	0.9157	35	0.0822	28	0.1206	31	0.0985	33	-1.47	0.0558	30
Portsmouth	0.028	0.9362	18	0.9827	18	0.0457	21	0.0332	24	0.0005	18	-0.21	0.0253	23
Robert Gordon	0.014	0.9101	21	0.9101	36	0.0899	30	0	1	0	1		0	1
Sheffield Hallam	0.037	1	1	1	1	0	1	0	1	0	1		0	1
Staffordshire	0.019	1	1	1	1	0	1	0	1	0	1		0	1
Sunderland	0.019	0.9398	17	0.9808	19	0.0602	24	0.0389	26	0.0418	27	-2.67	0.1284	39
Teesside	0.021	0.7879	36	0.9528	27	0.2121	38	0.2007	36	0.1731	36	-7.50	0.1107	38
Thames Valley	0.019	0.5106	40	0.9729	23	0.4894	40	0.3948	40	0.4752	40	-10.39	0.0324	26
West of England	0.037	0.8007	34	0.9380	32	0.0083	14	0.0040	19	0.0280	24	-0.96	0.0275	24
Westminster	0.026	0.8613	26	0.9377	33	0.1387	35	0.1363	33	0.0815	31	-1.99	0.0692	33
Wolverhampton	0.028	0.9177	20	0.9269	34	0.0823	29	0.0116	22	0.0099	21	-5.65	0.0197	21
Mean		0.8979		0.9669		0.0698		0.0600		0.0578		-4.59	0.0325	
Number on frontier		11		11		12		17		17			17	
Correlations: TE						-0.8314		-0.8354		-0.8485			-0.2795	
$C_{F, CRS}$								0.9205		0.9452			0.3651	
$C_{F, VRS}$										0.9705			0.4113	

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Table 1. Annual mean TE scores for alternative models

	TE (UAM)	TE (WAM)	Difference	Min	SD	No. on frontier
Model 1						
1995/6	0.927	0.938	0.011	0.493	0.118	19
1996/7	0.862	0.875	0.012	0.487	0.144	13
1997/8	0.864	0.875	0.011	0.527	0.134	13
1998/9	0.864	0.885	0.021	0.428	0.147	13
1999/0	0.895	0.908	0.013	0.577	0.129	20
2000/1	0.899	0.919	0.020	0.430	0.146	20
2001/2	0.893	0.906	0.014	0.483	0.132	14
2002/3	0.919	0.927	0.009	0.597	0.099	16
2003/4	0.884	0.887	0.003	0.531	0.118	11
Model 2						
1995/6	0.894	0.899	0.006	0.586	0.123	10
1996/7	0.811	0.813	0.002	0.497	0.164	10
1997/8	0.849	0.853	0.004	0.580	0.132	9
1998/9	0.897	0.904	0.006	0.477	0.107	9
1999/0	0.908	0.910	0.002	0.722	0.100	17
2000/1	0.918	0.925	0.008	0.526	0.103	15
2001/2	0.904	0.909	0.005	0.536	0.111	14
2002/3	0.915	0.916	0.001	0.616	0.096	15
2003/4	0.898	0.894	–0.004	0.511	0.104	11
Model 3						
1995/6	0.906	0.910	0.004	0.594	0.111	14
1996/7	0.870	0.873	0.004	0.501	0.137	11
1997/8	0.816	0.816	0.000	0.580	0.140	7
1998/9	0.900	0.906	0.006	0.546	0.099	8
1999/0	0.922	0.923	0.002	0.743	0.083	17
2000/1	0.910	0.916	0.006	0.519	0.107	15
2001/2	0.898	0.903	0.005	0.545	0.111	14
2002/3	0.926	0.926	0.000	0.602	0.091	15
2003/4	0.892	0.888	–0.004	0.485	0.108	11

Table 2. Cooper's congestion scores for Model 2

All universities					Congested universities	
	$\overline{C}_C$ (UAM)	$\overline{C}_C$ (WAM)	Difference	SD	Number	$\overline{C}_C$ (UAM)
1995/6	0.0501	0.0483	−0.0018	0.065	24	0.0857
1996/7	0.0521	0.0536	0.0015	0.060	28	0.0763
1997/8	0.0435	0.0455	0.0020	0.062	24	0.0743
1998/9	0.0441	0.0476	0.0035	0.056	27	0.0670
1999/0	0.0275	0.0276	0.0001	0.039	22	0.0513
2000/1	0.0423	0.0427	0.0004	0.053	23	0.0754
2001/2	0.0304	0.0301	−0.0003	0.035	22	0.0566
2002/3	0.0201	0.0178	−0.0023	0.032	20	0.0401
2003/4	0.0325	0.0334	0.0010	0.040	23	0.0564
Mean	0.0381	0.0385	0.0004			0.0648

Table 3. Results from Tone's approach and comparison with Cooper's approach (Model 2, unweighted)

All universities				Congested universities				
	$\overline{C}_C$	$\overline{C}_T$	Difference	Number	$\overline{C}_T$	$\overline{\rho}$	Max	Min
1995/6	0.0501	0.0486	−0.0015	24	0.0830	−7.87	−60.9	−0.25
1996/7	0.0521	0.0792	0.0271	27	0.1202	−10.61	−30.8	−0.59
1997/8	0.0435	0.0436	0.0001	24	0.0745	−4.41	−24.3	−0.71
1998/9	0.0441	0.0492	0.0051	26	0.0776	−8.53	−45.8	−0.82
1999/0	0.0275	0.0345	0.0070	22	0.0643	−17.57	−262.4	−1.63
2000/1	0.0423	0.0429	0.0006	23	0.0766	−12.85	−176.5	−0.10
2001/2	0.0304	0.0549	0.0245	22	0.1023	−3.99	−11.6	−0.82
2002/3	0.0201	0.0423	0.0222	19	0.0890	−7.65	−42.3	−0.32
2003/4	0.0325	0.0578	0.0253	23	0.1006	−4.59	−11.0	−0.21
Mean	0.0381	0.0503	0.0123		0.0876	−8.68		

Table 4. Results from Färe's approach and comparison with approaches of Cooper and Tone (Model 2, unweighted, all universities)

	$\overline{C}_{F, VRS}$	$\overline{C}_{F, VRS} - \overline{C}_T$	$\overline{C}_{F, VRS} - \overline{C}_C$	$\overline{C}_{F, CRS}$	$\overline{C}_{F, CRS} - \overline{C}_T$	$\overline{C}_{F, CRS} - \overline{C}_C$
1995/6	0.0604	0.0118	0.0103	0.0722	0.0236	0.0220
1996/7	0.0880	0.0088	0.0359	0.1281	0.0489	0.0760
1997/8	0.0512	0.0076	0.0076	0.0622	0.0185	0.0186
1998/9	0.0484	−0.0008	0.0042	0.0645	0.0153	0.0204
1999/0	0.0407	0.0062	0.0132	0.0455	0.0110	0.0179
2000/1	0.0450	0.0020	0.0027	0.0454	0.0024	0.0031
2001/2	0.0570	0.0021	0.0266	0.0635	0.0086	0.0332
2002/3	0.0336	−0.0087	0.0136	0.0450	0.0027	0.0249
2003/4	0.0600	0.0021	0.0275	0.0698	0.0119	0.0373
Mean	0.0538	0.0035	0.0157	0.0662	0.0159	0.0282

Table 5. Correlations: Model 2, n = 367

	C_T	C_C	$C_{F, CRS}$
C_C	0.441		
$C_{F, CRS}$	0.890	0.485	
$C_{F, VRS}$	0.904	0.464	0.855

Table 6. Percentage contribution of each input to congestion in congested universities

	Other expenditure	Academic staff	Postgrads	Undergrads	Number congested	$\overline{C}_C$ (UAM)
Model 1						
1995/6	19.7	33.3	13.1	33.9	17	0.0612
1996/7	17.0	33.9	20.0	29.1	24	0.0756
1997/8	11.6	31.8	36.5	20.1	23	0.0719
1998/9	5.9	41.9	23.7	28.5	25	0.0686
1999/0	17.6	29.2	7.2	46.0	19	0.0655
2000/1	16.6	24.9	10.8	47.7	16	0.0549
2001/2	25.7	18.7	19.0	36.6	21	0.0497
2002/3	20.1	35.7	8.5	35.7	18	0.0503
2003/4	22.2	27.8	17.4	32.6	22	0.0744
Mean	17.4	30.8	17.4	34.5		0.0636
Model 2						
1995/6	14.0	39.5	15.0	31.5	24	0.0857
1996/7	20.5	27.7	14.9	36.9	28	0.0763
1997/8	14.3	26.1	35.6	24.0	24	0.0743
1998/9	8.9	36.1	34.0	21.0	27	0.0670
1999/0	30.9	23.8	15.6	29.7	22	0.0513
2000/1	18.5	20.6	26.3	34.6	23	0.0754
2001/2	26.8	15.4	20.5	37.3	22	0.0566
2002/3	28.8	15.0	13.7	42.5	20	0.0401
2003/4	16.0	29.5	12.3	42.3	23	0.0564
Mean	19.8	26.0	20.9	33.3		0.0648
Model 3						
1995/6	11.8	33.9	19.1	35.2	23	0.0758
1996/7	21.8	20.0	20.9	37.3	23	0.0582
1997/8	15.7	21.5	34.6	28.2	26	0.0938
1998/9	14.4	29.4	33.8	22.4	27	0.0735
1999/0	43.4	11.4	18.3	26.9	21	0.0396
2000/1	17.2	21.7	26.1	34.9	22	0.0747
2001/2	33.6	11.6	18.3	36.4	24	0.0511
2002/3	32.6	21.9	11.5	33.9	19	0.0477
2003/4	14.6	40.3	11.1	34.0	23	0.0636
Mean	22.8	23.5	21.5	32.1		0.0642

Table 7. Disaggregation of Cooper's congestion score for each congested university (Model 2, 2003/4)

	Other expenditure	Academic staff	Postgrads	Undergrads	C_C
Anglia	0.0321			0.2175	0.0624
Bournemouth				0.1279	0.0320
Brighton	0.0177			0.0176	0.0088
Central England		0.0954		0.2423	0.0844
Derby				0.0734	0.0184
East London			0.3681		0.0920
Glamorgan				0.0651	0.0224
Glasgow Caledonian		0.3351		0.2128	0.1370
Hertfordshire	0.0924			0.0633	0.0389
Kingston				0.0399	0.0100
London Metropolitan		0.0495	0.1443	0.0397	0.0584
London South Bank		0.1498		0.0400	0.0474
Napier		0.1776			0.0444
Northumbria	0.1761	0.1304			0.0766
Nottingham Trent		0.0984	0.0679	0.2181	0.0961
Plymouth	0.1226			0.1006	0.0558
Portsmouth		0.1012			0.0253
Sunderland	0.3026		0.0300	0.1811	0.1284
Teesside	0.0132	0.1712		0.2584	0.1107
Thames Valley	0.0027			0.1267	0.0324
West of England		0.0794		0.0306	0.0275
Westminster	0.0691	0.1197	0.0274	0.0607	0.0692
Wolverhampton				0.0786	0.0197
Mean	0.0360	0.0666	0.0277	0.0954	0.0564

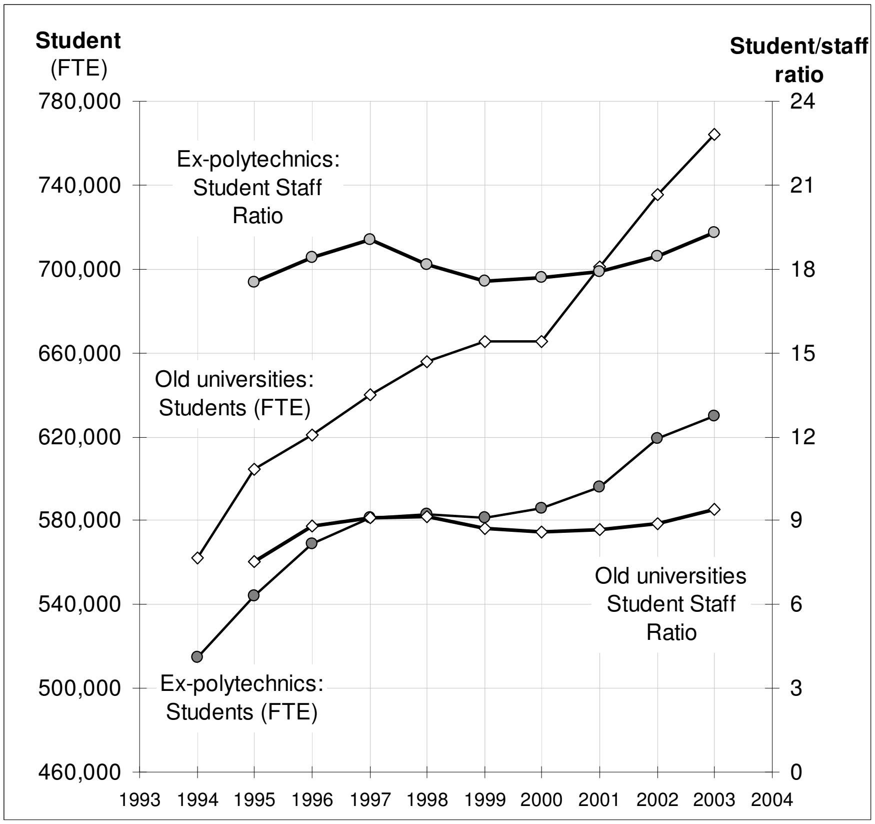

Figure 1. Students and staff: old and new universities

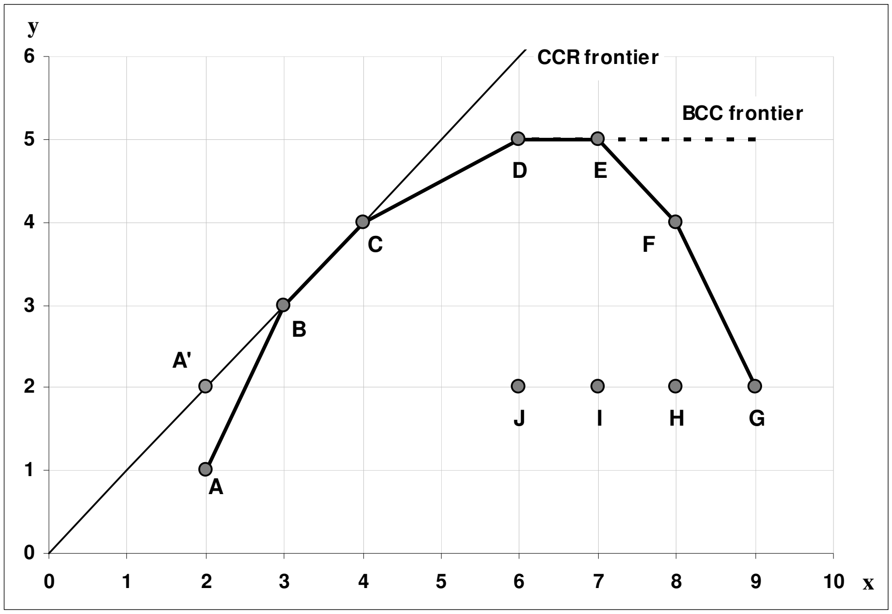

Figure 2. DEA models and congestion

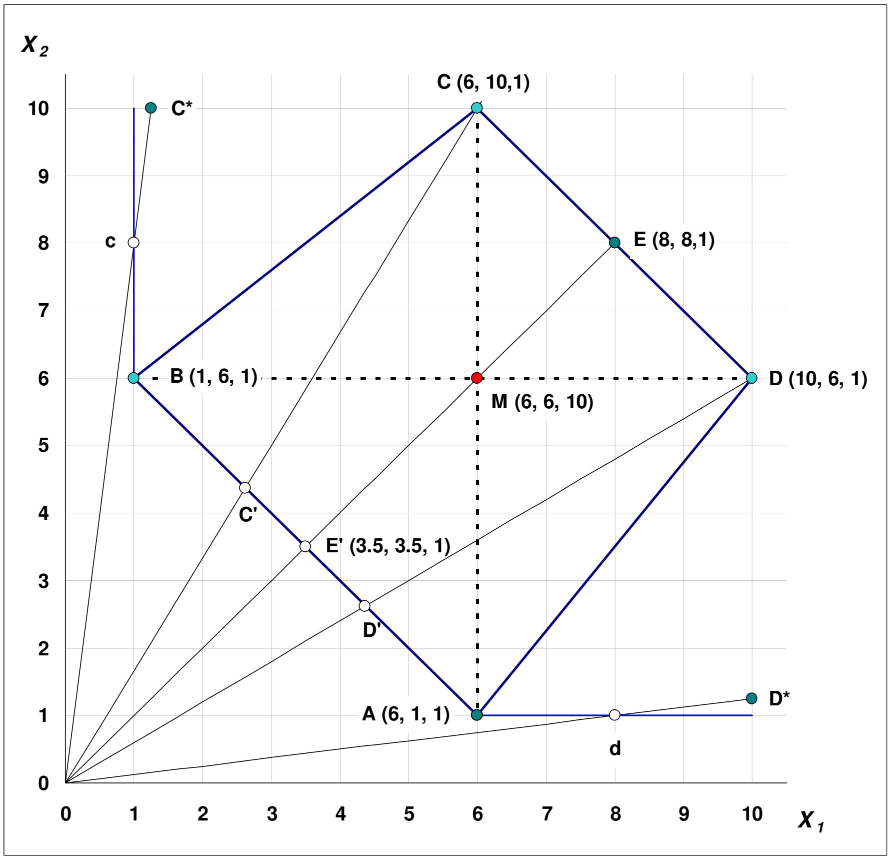

Figure 3. An illustrative example

Erratum: M (6, 6, 5) not M (6, 6, 10)

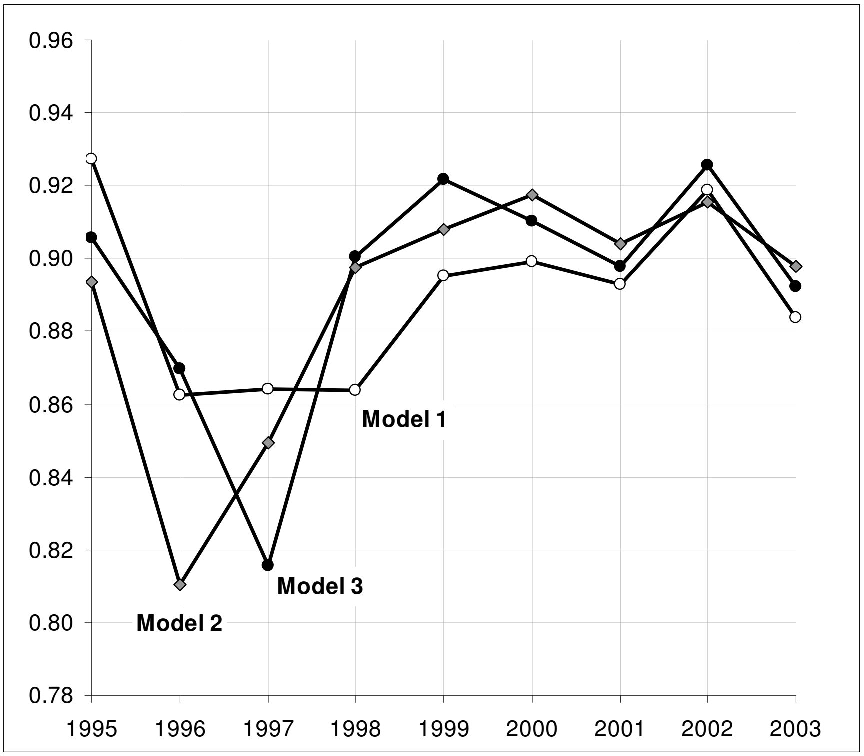

Figure 4. Technical efficiency

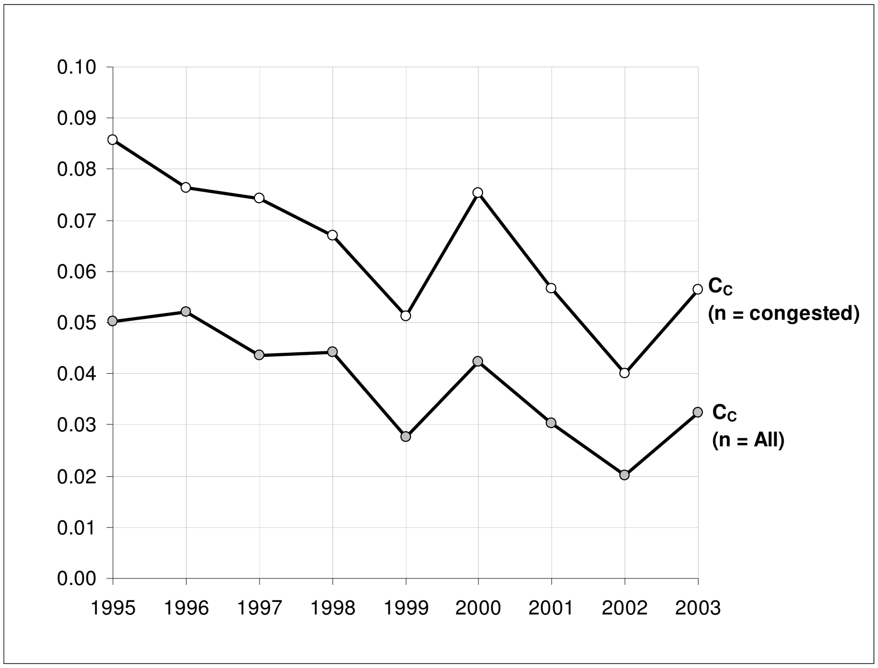

Figure 5. Cooper's measure of congestion

Erratum: C_C needs a bar on the C.

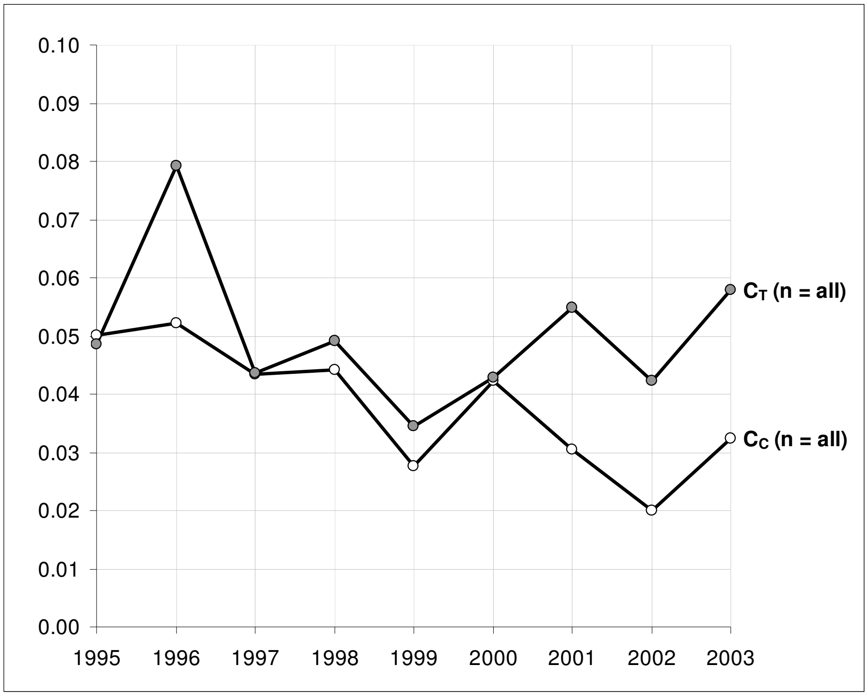

Figure 6. Tone and Cooper's measures of congestion

Erratum: C_T and C_C need a bar on the C.

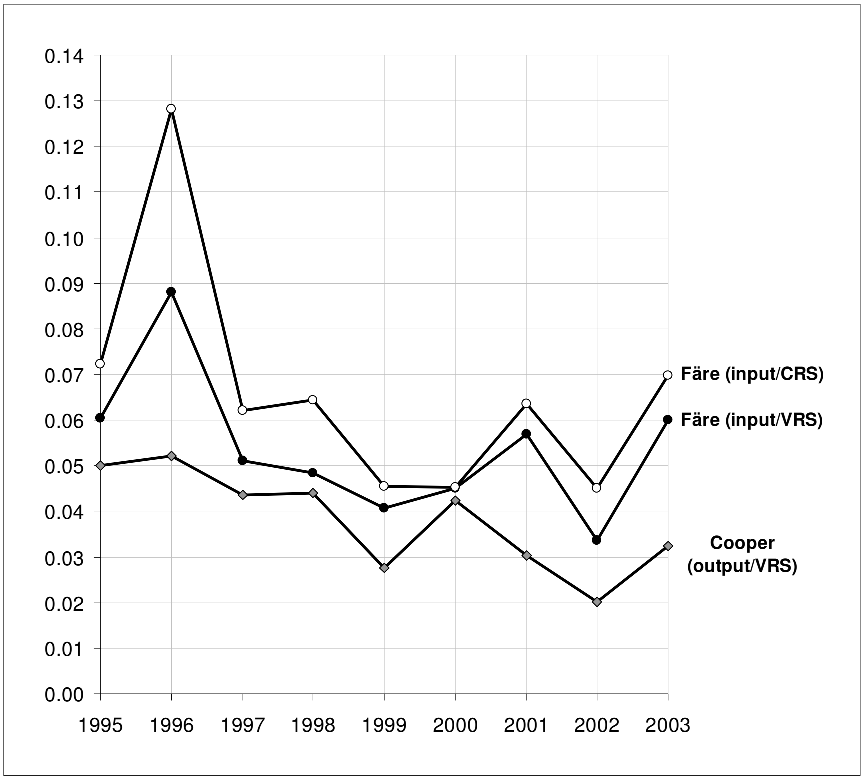

Figure 7. Färe and Cooper's measures of congestion

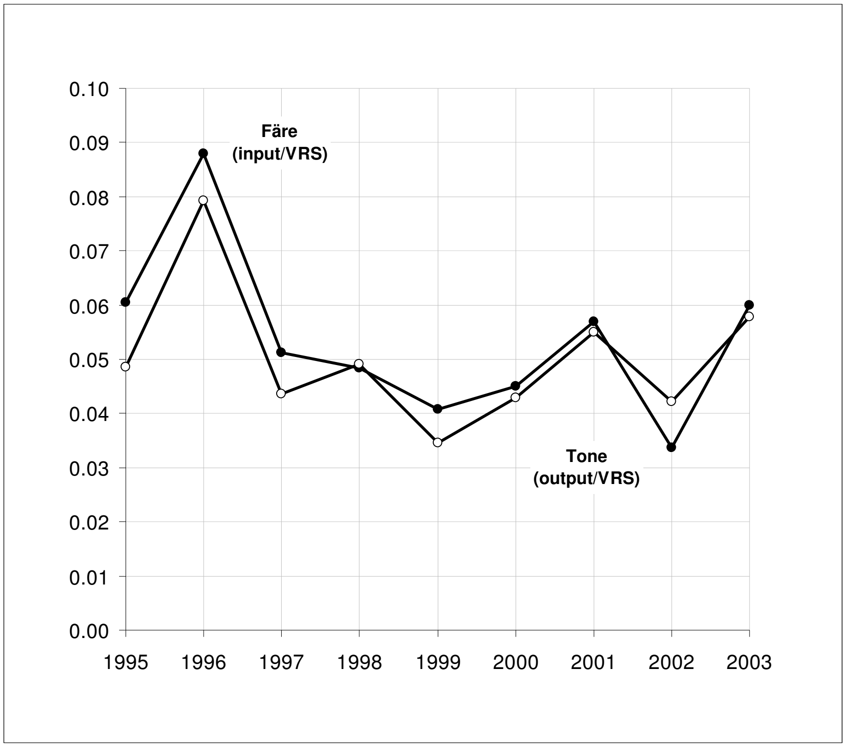

Figure 8. Färe and Tone's measures of congestion