Concentration in the International Arms Industry*

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Abstract

The end of the Cold War led to a large drop in world military expenditure, rising fixed costs of developing weapons because of technological changes and a reduction of national preference for domestic weapons. Alongside these developments has been an increase in concentration in the world arms industry, which at the end of the Cold War had been very unconcentrated with concentration ratios close to the Sutton lower bound. This paper provides an empirical and theoretical analysis of this process. It examines the dynamics of the evolution of concentration and then shows that a trade model with optimal procurement decisions can capture the main features of this empirical analysis.

JEL: F12, H56, L10

Keywords: market structure, arms industry, procurement

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1 Introduction.

The end of the Cold War saw fundamental changes in the international security environment that led to large reductions in military spending worldwide, with important implications for the international arms industry. At the same time changes in government behaviour and attitudes to arms production and changes in weapons technology were impacting upon the producers. The result was substantial turmoil and restructuring, the main observable outcome of which was the increase in concentration in the industry. There is, however, little detailed empirical research on the changes that have taken place in the industry and no attempts to identify what are the most important factors driving the process of change.

The changes in military spending involved were certainly substantial. The antagonisms of the Cold War had seen military expenditures peak at $1360 billion in the middle 1980s, fall gradually at first with improving East-West relations, then fall rapidly with the disintegration of the Soviet Union to $823 billion (constant 1997 prices). It is estimated that world military expenditure fell at about 6% per annum in real terms over the decade 1987-97, 7% a year in the developed world and 1% a year in the developing world, with the most dramatic fall was in the former Soviet Union.1 The arms trade dropped by a half between the 1987 all time high of $81 billion and the 1994 trough of $42 bn (in 1997 prices), rising to $55bn in 1997. The Asian crisis of 1997 subsequently hit arms sales, since this was an area where demand had been strong. Procurement of weapons also fell sharply, with SIPRI (2000) estimating that arms production (domestic demand plus exports minus imports) in 1997 was 56% of its 1987 level in the US, 77% in France and 90% in the UK.

This decline in the demand for armaments has been associated with an increase in both concentration and competition in the world arms industry, initially in the US but followed by the rest of the world.2 The outcome of these processes reflected not simply the change in demand, but also the unique characteristics of the industry, when compared to civil production, and how these changed over the period. While production for the military

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1 BVC(2000), previously ACDA, the US Arms Control and Disarmament Agency.
2 As SIPRI (2001, p302) comment: “A period of intensive mergers and acquisitions (M&A) began in the early 1990s. Among large aerospace companies, concentration culminated in 1997-8 in the USA and in 1999-2000 in Western Europe.”
is not homogeneous major weapons systems are the main products for leading arms producers and have particular characteristics that have over the years led to particular corporate structures. They involve high fixed R&D costs financed by the governments and fairly short production runs with steep learning curves (Sandler and Hartley, 1995, chapter 7). This means that average costs fall sharply with each further unit produced and so major weapons producers can gain economies of scale and their minimum efficient scale is large relative to the size of the market. This led to production being concentrated in relatively few states (Dunne, 1995).

Traditionally, because the state, which had strong national preferences, was the customer, major countries largely relied on their domestic defence industries and while most manufacturing industries went multinational, the arms industry remained national. Smaller countries which could not afford the large fixed costs imported major weapons systems. With the fall in demand, the ability of even the major countries to maintain a domestic defence industrial base was called into question, making them more willing to import. As a result domestic and foreign weapons would appear to be regarded as closer substitutes than in the past. This willingness to import has also led to increased competition, which helps keep down prices and stimulates innovation by firms and this can clearly be considered of benefit of governments. There is, however, the problem that the drop in demand can also drive firms below their minimum efficient scale. In addition, in response to cost pressures, arms producers have increasingly been using components that are commercial ‘off-the-shelf’ (COTS) products, produced by manufacturers who would not see themselves as part of the arms industry, with important implications for the structure of the industry.

In this new environment, Governments have also had to decide whether to allow mergers and acquisitions that would reduce competition and in particular whether to allow

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3 It consists of a whole range of products, from small arms to the large complicated weapon systems, as well as material that is not directly military.

4 In contrast to small arms production, which is relatively standard and widely dispersed.

5 This tension between the benefits of scale and the benefits of competition has in fact been the central defence industrial policy dilemma for the last 40 years. A discussion of the structure at the end of the Cold War can be found in Smith (1990) a model of the process of competition can be found in Garcia (1999).

6 The factors driving up military costs, particularly the fixed costs of R&D, are analysed in Kirkpatrick (1995).
mergers and acquisitions that involved foreign partners. The most striking change in industrial policy was in the US. In 1993 a merger wave was stimulated by the ‘last supper’ when the Pentagon Deputy Secretary Perry told a dinner of defence industry executives that they were expected to start merging. It ended when the Pentagon decided it had gone far enough and blocked the merger of Lockheed Martin with Northrop Grumman in early 1997 (Markusen and Costigan, 1999). In Europe the process was more complicated, since restructuring would involve cross-border mergers, raising political issues, and the major players had quite different ownership structures, including a substantial degree of state ownership in France. Both factors made a financially driven merger boom of the US type much more difficult. But it is clear that it is the decisions of governments played a vital role in changing the structure of the industry, and this is a key feature of the theoretical model we develop below.

Clearly, the changing nature of the international arms market is the result of a number of factors on both the demand and supply side, interacting in an apparently complex manner. This paper makes a contribution to developing an understanding of the processes at work by identifying the most significant factors, through a detailed analysis of the changing structure of the international market, using data on the major international arms companies and by developing a model consistent with the stylised facts and findings of the empirical analysis.

In the next section the evolution of concentration in the defence industry in the period 1990-98 is described. An analysis of the size and growth of firms is then undertaken to see whether there has been any systematic change in the size of surviving firms over this period. Section 3 then sets out a trade model with optimal procurement decisions, which is then calibrated using the findings of the empirical analysis. This calibrated model then provides a plausible explanation of the observed changes in concentration in terms of falling demand and rising capital and R&D costs. Section 4 presents some conclusions.
2 Empirical Analysis

2.1 Concentration

With the decline in demand for arms after the Cold War, companies were forced to consider their corporate strategies. Firms had a choice between five options on a civil-military axis: converting their military production facilities to civil production; diversify, by growing or acquiring civil businesses; divest their defence businesses; cooperate through joint ventures; or concentrate on defence, acquiring the defence businesses others divested. Apart from conversion, a route few firms followed, the other strategies were widely adopted by different firms. The effect of these corporate choices within the constraints imposed by national governments can be seen in Table 1, which describes the evolution of the industry 1990-1998. The data set we use is the SIPRI arms company database, described in a data appendix.

Table 1. The defence industry 1990-1998. Number of firms, \( N \), Total arms sales, \( AS \), Inverse Herfindahl, \( IH \), Concentration Ratios, 5 firm \( C5 \), 10 firm, \( C10 \), 20 firm, \( C20 \).

<table>
<thead>
<tr>
<th></th>
<th>1990</th>
<th>1998</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>US</td>
</tr>
<tr>
<td>( N )</td>
<td>123</td>
<td>51</td>
</tr>
<tr>
<td>( AS )</td>
<td>191680</td>
<td>114057</td>
</tr>
<tr>
<td>( IH )</td>
<td>49</td>
<td>24.41</td>
</tr>
<tr>
<td>( C5 )</td>
<td>22</td>
<td>35</td>
</tr>
<tr>
<td>( C10 )</td>
<td>36</td>
<td>56</td>
</tr>
<tr>
<td>( C20 )</td>
<td>55</td>
<td>78</td>
</tr>
</tbody>
</table>

Table 1 gives the number of firms in the sample, their total arms sales and various measures of concentration. Sales by all the firms fell by 20% over the period, 24% in the US and 11% in the rest of the world. The average size of firm in the sample was relatively stable at about $2,200m in the US and $1,100 in the rest of the world. This is of course a changing set of firms, and Table 2 shows the pattern of entry and exit.

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\(^7\)Smith (2001) discusses the evolution of the industry and the corporate responses in more detail.
Table 2, Change in number of firms 1990-1998.

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>US</th>
<th>nonUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>123</td>
<td>51</td>
<td>72</td>
</tr>
<tr>
<td>exits</td>
<td>29</td>
<td>18</td>
<td>11</td>
</tr>
<tr>
<td>survivors</td>
<td>94</td>
<td>33</td>
<td>61</td>
</tr>
<tr>
<td>entrants</td>
<td>10</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>1998</td>
<td>104</td>
<td>39</td>
<td>65</td>
</tr>
</tbody>
</table>

All the measures show increased concentration. The inverse Herfindhal can be interpreted as an equivalent number of identical firms and we shall use it for that purpose below. A convenient standard with which to judge the degree of concentration in the arms industry is provided by Sutton (1998) who suggests an approach, which does not try to predict a unique equilibrium for the game or the whole size distribution, but to provide a lower bound on the concentration that one might observe in the market. It is based on the assumption that any observed industry is built up from a range of sub-markets. In the international arms industry the sub-markets are defined by the various types of weapons from particular countries. He shows that certain basic principles, (e.g. firms make enough profits to cover their fixed costs and no viable sub-market will be left unexploited) provide restrictions on the set of Nash equilibria, and these together with fairly weak conditions on whether incumbents or entrants will enter a new sub-market opportunity provide a lower bound on concentration. This is given by

$$c_k = \frac{k}{n} \left[ 1 - \ln\left(\frac{k}{n}\right) \right]$$

where $c_k$ is the lower bound for the $k$ firm concentration ratio and $n$ is the total number of firms in the sample.

As Table 1 shows the five largest companies in the SIPRI list accounted for almost 22% of global arms production in 1990. This is very close to Sutton’s independent sub-market lower bound for the five firm concentration ratio given above, which is 20%. Similarly the 10 and 20 firm ratios are close to their lower bounds. At the end of the Cold War the international arms industry as a whole was not very concentrated. In fact, concentration in total sales of these companies was higher than in arms sales even though the commercial markets these firms were operating in were very different. In 1994, the five...
largest arms firms accounted for 28% of the total, 1998 41% of the total, in 1999 43% of the total. This large increase in the share of the top companies is continued further down the sizes, as shown for the largest 10 and 20 and indeed across the distribution. The sub-markets were, as one would expect more concentrated, and the increase in concentration was greater for the US, where more firms also exited. It seems likely that by its nature major weapons systems would naturally be a very concentrated market like pharmaceuticals, civil airliners, etc., but that the role of the national governments in attempting to maintain national defence capabilities has been to inhibit increasing concentration.

2.2 Growth of firms

To analyse the growth of the surviving firms, we use the standard equation relating log arms sales, $A_{it}$ in year $t = 1998$ to log size in $t - 1 = 1990$

$$\ln A_{it} = \mu + \rho \ln A_{it-1} + \varepsilon_{it};$$

$$E(\varepsilon_{it}) = 0; \quad E(\varepsilon_{it}^2) = \sigma_\varepsilon^2.$$  

Gibrat’s law is that $\rho = 1$ so that growth in size is random. This generates a log-normal distribution with increasing variance, see for instance Sutton (1997), Dunne and Hughes (1994) and Hart and Oulton (1996). In 1990 log $A_i$ is close to normal, Jarque-Bera tests have p values of 0.054 for all firms, 0.199 for US and 0.132 for others. However, by 1998 the distributions are clearly non-normal with p values of 0.000, 0.003 and 0.031 for all, US, others. As observations for both 1990 and 1998 are needed for this regression only the companies that survive for the whole period and do not have missing values for the relevant variable in either year are included\textsuperscript{8}. Assuming that $\ln A_{it-1}$ is uncorrelated with $\varepsilon_{it}$, this gives a relationship determining the evolution of the variance of log firm size, a measure of concentration:

$$\sigma_t^2 = \rho^2 \sigma_{t-1}^2 + \sigma_\varepsilon^2.$$  

\textsuperscript{8}The results are robust to correction for sample selection bias. This is because it is difficult to predict which firms exit from this data. The only significant difference is US firms are more likely to exit. Initial size is not significant predictor for either group.
Noting that $R^2 = 1 - (\sigma^2_t / \sigma^2)$, this implies that whether variance is increasing or decreasing is determined by whether the ratio:

$$\frac{\sigma^2_t}{\sigma^2_{t-1}} = \frac{\rho^2}{R^2}$$

is greater or less than unity. For $\rho = 1$, this is always greater than unity as long as $\sigma^2_\varepsilon \neq 0$, the variance does not converge to an equilibrium but continues to increase through time. For $\rho < 1$, the equilibrium variance is

$$\sigma^2_\infty = \frac{\sigma^2_\varepsilon}{1 - \rho^2}.$$ 

Estimates of the equation for the whole sample and for US and non-US firms are given in Table 3.

Table 3. Estimates for surviving companies, robust standard errors in parentheses, standard error of regression and maximised log-likelihood.

<table>
<thead>
<tr>
<th></th>
<th>$N$</th>
<th>$\mu$</th>
<th>$\rho$</th>
<th>$\sigma_\varepsilon$</th>
<th>$R^2$</th>
<th>$MLL$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>94</td>
<td>1.41</td>
<td>0.77</td>
<td>0.67</td>
<td>0.60</td>
<td>-94.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.58)</td>
</tr>
<tr>
<td>US</td>
<td>33</td>
<td>0.04</td>
<td>0.94</td>
<td>0.84</td>
<td>0.60</td>
<td>-40.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(1.11)</td>
</tr>
<tr>
<td>nonUS</td>
<td>61</td>
<td>1.75</td>
<td>0.73</td>
<td>0.54</td>
<td>0.65</td>
<td>-48.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.53)</td>
</tr>
</tbody>
</table>

Gibrat’s law $\rho = 1$ is not rejected in the US sample, though it is in the rest of the world and the pooled sample, though pooling the two samples is strongly rejected. For the rest of the world, the estimates indicate that small firms grow faster than large firms. The standard error of regression is substantially larger in the US than the rest of the world. For the rest of the world the estimates imply an equilibrium standard error of 1.16. However the estimates for the rest of the world are sensitive to outliers, in particular two small arms producers (GKN and Celsius) which grew from less than $200m to over $1000m by acquisition. Excluding the very small firms with arms sales less than $400m, Gibrat’s law holds almost exactly for the non-US sample as well as the US sample. This suggests that
growth of firms is random\(^9\) and that in explaining the evolution of the market, we should look at features that are particular to the market, that is characteristics of demand, rather than to features that are particular to individual firms. In the next section, we provide a theoretical model of the evolution of the industry which emphasises characteristics of demand.

### 3 A Model of Optimal Procurement and Trade

#### 3.1 The Model

This section sets out a model of the global arms market which can capture the main features discussed in the previous section. We consider a very specific problem: the procurement by a military authority from a private military sector given a fixed budget. We assume away asymmetric information and the need for incentive mechanisms to address this problem.\(^{10}\)

We consider an international market for arms consisting of \(\ell\) countries where country 1 produces differentiated goods \(j = 1, 2, \ldots, n_1\), country 2 produces goods \(j = n_1 + 1, n_1 + 2, \ldots, n_1 + n_2\) etc, so there are \(\sum_{i=1}^{\ell} n_i = N\), say, goods in total. Each variety is produced by a single firm. The maximum quality of good \(j\) in country \(i = 1, 2, \ldots, \ell\) is \(q_{ij}\) which is the quality of the procured good. We assume that each firm can produce a lower quality good at the same cost and we allow for the possibility that there is an arms export regime in place that restricts the quality of the imported good by country \(i\) to \(u_{ij} \leq q_{kj}\), the quality procured by country \(k\) of variety \(j = n_{k-1} + 1, n_{k-1} + 2, \ldots, n_{k-1} + n_k\). We take this regime to be exogenously imposed on the military authority making the procurement decisions and we do not go into details of how this regime can be sustained.

It makes for a simpler presentation if we focus on decisions in country 1. Military authority 1 procures \(d_{1j}, j = 1, 2, \ldots, n_1\) domestic goods at quality \(q_{1j}\) and imports \(m_{1j}, j = n_1 + 1, n_1 + 2, \ldots, N\) military goods at quality \(u_{1j}\). The military capability production function for a particular weapon type in country 1 is assumed to take the form of a

\(^9\)We tried various other characteristics of firms at the beginning of the period, e.g. total sales (rather than just arms sales) and type of product, but none were significant.

\(^{10}\)Rogerson (1991) discusses incentive mechanisms in a military procurement context.
Dixit-Stigliz CES production function given by

\[ M_1 = \left[ w_1 \sum_{j=1}^{n_1} (q_{1j}d_{1j})^\alpha + (1 - w_1) \sum_{j=n_1+1}^{N} (u_{1j}m_{1j})^\alpha \right]^{\frac{1}{\alpha}} \quad ; \quad \alpha \in [-\infty, 1] \quad (1) \]

In (1) the weights \( w_1 \) and \( 1 - w_1 \), with \( w_1 \in \left[ \frac{1}{2}, 1 \right] \), express the security preferences for domestic rather than imported procurement in country 1 and \( \frac{1}{1-\alpha} \) is the elasticity of substitution between different varieties. A variety may be thought of as a type of weapon produced in a particular country by a single firm. Countries need a mix of weapons, hence the demand for variety, but different types of weapons can be quite close substitutes in destructive power. Power projection, as in Kosovo or Afghanistan, can be done by ground troops, long-range bombers, attack aircraft launched from carriers, cruise missiles launched from submarines, etc. In (1) there is of course diminishing marginal capability from increasing any one variety, and this can capture the insecurity associated with over-dependence on any one supplier. For this exercise, where the emphasis is on the supply side we are not modelling the determination of \( M \). In addition we are interested in the global market and we do not attempt to model either the demand for military expenditure or the decision to have an arms industry.\(^{11}\)

The manner in which quality enters into this form of the CES production function can be related to military operational analysis where the probability of country 1 defeating country 2 in a battle using only domestically produced varieties \( k \) and \( h \) respectively is usually modelled as

\[ p_{kh} = \frac{q_k}{q_h} \left( \frac{d_k}{d_h} \right)^\eta \]

where \( \eta \), known as the Lanchester (1916) coefficient, depends on the type of combat, see the discussion in Sandler and Hartley (1995). For dispersed individual duels it is unity, for battles between massed ranks, it is two. For cases where technological edge translates directly into victory, it is close to zero. Treating it as unity on average, as is done above by using \( q_jd_j \) seems a sensible simplification.

Let \( p_{1j} \) be the price of the procured domestic good and \( P_j \) be the price of the traded good of variety \( j \) produced by firms in all producing countries \( j = 1, 2, \ldots, N \). Then the

\(^{11}\) These aspects are treated in Levine and Smith (2000).
budget constraint for military authority 1 is:

\[ \sum_{j=1}^{n_1} p_{1j} d_{1j} + \sum_{j=n_1+1}^{N} P_j m_{1j} = G_1 \]  

(2)

where \( G_i \) is military expenditure in country \( i \). Defining \( N_i \) by

\[ N_i = n_1 + n_2 + \cdots + n_i \]

for \( i \geq 1 \) (in which case \( N_1 = n_1 \) and \( N_\ell = N \), country \( i = 1, 2, \cdots, \ell \) produces varieties \( j = N_{i-1} + 1, N_{i-1} + 2, \cdots, N_{i-1} + n_i = N_i \) and imports \( m_{ij} \) units of variety \( j = 1, 2, \cdots, N_{i-1}, N_i + 1, N_i + 2, \cdots, N \) (defining \( N_0 = 0 \)). It follows that the exports of variety \( j = 1, 2, \cdots, n_1 \) by country 1 are given by

\[ x_{1j} = \sum_{i=2}^{\ell} m_{ij} \]

(3)

where \( m_{ij} \) are the imports of variety \( j \) by country \( i \). The model is completed by specifying the following cost structure for the firm. Firm \( j \) produces \( d_j \) units of variety \( j \) for its domestic government at a price \( p_j \) and exports \( x_j \) units at a price \( P_j \). The cost of producing \( y_j = d_j + x_j \) of quality \( q_j \) is assumed to be

\[ C(y_j, q_j) = F + f q_j^\beta + c y_j ; \beta > 1 \]

(4)

The first term in (4) we associate with fixed capital costs other than R&D, the second term with fixed R&D costs and the final term constitutes variable costs.\footnote{Since there are many ingredients in this model and our main focus is on the endogenous determination of the number of firms, R&D is treated here in a rather rudimentary fashion ignoring, for example, uncertainty. There is a large literature that explores the relationship between R&D intensity and market concentration. Tischler (2001) is a recent contribution with a useful review of the issues.} It follows that the profit of this firm is

\[ \pi_j = p_j d_j + P_j x_j - C(y_j, d_j) \]

(5)

and since there is free entry and exit we must impose the participation constraint \( \pi_j \geq 0 \) on the procurement decision.

3.2 Optimal Policy in the Single Economy

We first consider the optimal decisions of a single military authority taking the decisions of other military authorities as exogenous. We assume that the arms export control regime is in force and limits the quality of arms that can be exported by a country to some
exogenous proportion of the maximum quality available. The sequencing of events is as follows:
1. Military authority 1 procures domestic goods of quantity \(d_{ij}\) and quality \(q_{ij}\) at price \(p_{ij}\) for \(j = 1, 2, \cdots, n_1\), subject to a non-negative profit participation constraint. To do this it must allocate its budget between \(n\) domestic procurement and imported goods on the basis of an expected world market price realised in the market equilibrium of the next stage. However at stage 1 there is only commitment to the domestic procurement decision.
2. In a Bertrand equilibrium of this stage of the game, firms in producer country \(i\) set world prices \(P_j\) for variety \(j = N_{i-1} + 1, N_{i-1} + 2, \cdots, N_{i-1} + n_i\), and export quantity \(x_{ij}\) at quality \(u_{kj} \leq q_{ij}\) to country \(k \neq i\).
3. Since stage 1 involves no commitment to import decisions,\(^{13}\) having set the budget for imports, \(G_1 - \sum_{mpj=1}^{n_1} p_{1j}d_{1j}\), on the basis of the expected equilibrium import price, military authority 1 may change the particular combination of imports of each good, \(m_{1j}, j = n_1 + 1, n_1 + 2, \cdots, N\) given their actual (possibly out-of-equilibrium) price \(P_j\) and quality \(u_{1j}\).

To solve for the equilibrium\(^{14}\) we proceed by backward induction starting at:

**Stage 3**
Given the price \(P_j\), the importing military authority 1 chooses \(m_{1j}\) to maximize \(M_1\) given by (1) subject to its budget constraint (2) where the procurement element is given. To carry out this optimization define a Lagrangian

\[
M_1 - \lambda \left( \sum_{j=n_1+1}^{N} P_j m_{1j} - \left[ G_1 - \sum_{j=1}^{n_1} p_{1j}d_{1j} \right] \right) \tag{6}
\]

\(^{13}\)Notice the dichotomy between the domestic procurement decision which involves commitment and the import procurement decision where the country does not commit at stage 1 and is a price-taker at stage 3. If the number of producers is small this might seem problematic unless we add a large fringe of non-producers who only import. Then each producer country becomes a small player when it comes to the import decision. For the case on monopolistic competition (see below) the model can be easily modified in this way without changing the basic results.

\(^{14}\)Note that in the absence of procurement and quality considerations the trade equilibrium corresponds exactly to the seminal ‘new-trade’ model of Krugman (1979) and the imperfect competition model set out in Beath and Katsoulacos (1991), chapter 3. Then stage 1 of our model is the free-entry process. With procurement each military authority by choosing the procurement price, in effect, chooses the number of domestic firms.
Then the first-order conditions\footnote{Details of second-order conditions are omitted throughout, but can be shown to hold provided } are:

\[
\frac{1}{\alpha} \left[ \sum_{j=1}^{N} M_{ij}^\alpha \right]^{1/\alpha - 1} \alpha(1 - w_i)u_{ij}^\alpha m_{ij}^{\alpha - 1} = \lambda P_j \quad ; \quad j = 1, 2, \cdots, N \tag{7}
\]

Then dividing the \( j \)th equation of (7) by the \( k \)th equation we have

\[
\left( \frac{u_{ij} m_{ij}}{u_{ik} m_{ik}} \right)^{\alpha - 1} = \frac{u_{ik} P_j}{u_{ij} P_i} \tag{8}
\]

and substituting back into (2) we get

\[
\sum_{k=n_1+1}^{N} P_j u_{ik} m_{ik} \left( \frac{u_{ik} P_j}{u_{ij} P_i} \right)^{1/\alpha - 1} = G_1 - \sum_{j=1}^{n_1} p_{1j} d_{1j} \tag{9}
\]

which results in the demand by military authority 1 for good \( j \), \( j = n_1 + 1, n_1 + 2, \cdots, N \) given by

\[
m_{1k} = \frac{G_1 - \sum_{j=1}^{n_1} p_{1j} d_{1j}}{u_{ik} \left( \frac{P_j}{u_{ij}} \right)^{\sigma} \sum_{j=n_1+1}^{N} \left( \frac{P_j}{u_{ij}} \right)^{1-\sigma}} \tag{10}
\]

where \( \sigma = \frac{1}{1-\alpha} > 1 \). For any country \( i = 1, 2, \cdots, \ell \) import demand for any good \( j = 1, 2, \cdots, N \) of quality \( u_{ij} \) can be conveniently written as

\[
m_{ij} = \frac{\left( G_i - \sum_{j=n_i-1+1}^{n_i} p_{ij} d_{ij} \right)}{u_{ij} \left( \frac{P_j}{u_{ij}} \right)^{\sigma} \sum_{k \neq j}^{N} \left( \frac{P_k}{u_{ik}} \right)^{1-\sigma}} \quad ; \quad j \neq N_i - 1 + 1, N_i - 1 + 2, \cdots, N_i - 1 + n_i \quad \text{if } j = N_i - 1 + 1, N_i - 1 + 2, \cdots, N_i - 1 + n_i \tag{11}
\]

Then total demand on the world market for good \( j \) is given by \( \sum_{i=2}^{\ell} m_{ij} \).

To interpret and manipulate (11) it is convenient to define

\[
\hat{P}_i = \sum_{k \neq [N_i-1, N_i]}^{N} \left( \frac{P_k}{u_{ik}} \right)^{1-\sigma} = \hat{P}_i^{1-\sigma} \tag{12}
\]

Then \( \hat{P}_i = \hat{P}_i^{1-\sigma} \) is a quality-adjusted version of the familiar price index of imported goods by country \( i \) used in the product differentiation literature (see, for example, Beath and Katsoulacos (1991 chapter 3). Let \( G_{mi} = G_i - \sum_{j=N_i-1+1}^{N_i} p_{ij} d_{ij} \) be the part of the military budget devoted to imports. Then (11) can be written

\[
m_{ij} = \frac{u_{ij}^{\sigma-1} G_{mi}}{P_j^{\sigma} \hat{P}_i} \tag{13}
\]
The importance of (13) is that the elasticity of demand for variety \( j \) on the world market with respect to price and quality are constant at elasticities \(-\sigma\) and \(\sigma - 1\) respectively.

**Stage 2**

In country 1, firm \( j = 1, 2, \cdots, n_1 \) is required by the procuring authority to produce quality \( q_{1j} \geq u_{kj} \), the quality exported to country \( k \). Then profit at stage 2 is given by

\[
\pi_{1j} = (p_{1j} - c)d_{1j} + (P_j - c)x_{1j} - f q_{1j}^\beta; \ j = 1, 2, \cdots, n_1 \tag{14}
\]

where exports are given by

\[
x_{1j} = \sum_{i=2}^\ell m_{ij} = \sum_{i=2}^\ell \frac{u_{ij}^\sigma - 1 G_{mi}}{P_j^\sigma \tilde{P}_i} \tag{15}
\]

Maximizing with respect to \( P_j \) gives the first-order conditions

\[
(P_j - c) \frac{\partial x_{1j}}{\partial P_j} + x_{1j} = 0 \tag{16}
\]

where from (13)

\[
\frac{\partial x_{1j}}{\partial P_j} = -\frac{\sigma x_{1j}}{P_j} - P_j^{-\sigma} \sum_{i=2}^\ell \frac{u_{ij}^\sigma - 1 G_{mi}}{P_j^\sigma \tilde{P}_i} \left( \frac{\partial \tilde{P}_i}{\partial P_j} \right) \tag{17}
\]

strategic interaction term

In working out the effect of a change in the price of variety firm \( j \) considers two effects: the first term takes the total price index of imports facing other countries \( \tilde{P}_i; i = 2, 3, \cdots, \ell \) as given. The second *strategic* term considers the effect on each of these price indices of the firms export price. We bypass the complications raised by this term by adopting the standard assumption of monopolistic competition where there a sufficient number of firms to justify ignoring this effect. Then substituting (17) back into (16), the first order condition becomes

\[
\left[ -\frac{\sigma(P_j - c)}{P_j} + 1 \right] x_{1j} = 0; \ j = 1, 2, \cdots, n_1 \tag{18}
\]

Hence using (15) and imposing symmetry between products from the same country, we obtain from (18) the Lerner Index for any variety \( j \in [1, n_1] \) in country 1 as

\[
L_1 = \frac{P_1 - c}{P_1} = \frac{1}{\sigma} \tag{19}
\]

Similarly for variety \( j = N_{i-1} + 1, N_{i-1} + 2, \cdots, N_{i-1} + n_i \) in country \( i \) we have

\[
L_i = \frac{P_i - c}{P_i} = \frac{1}{\sigma} \tag{20}
\]
(20) for \(i = 1, 2, \ldots, \ell\) gives \(\ell\) equations in \(\ell\) prices, one for each country. This is the Bertrand equilibrium at stage 2 of the game in the absence of strategic interaction.

**Stage 1**

Imposing symmetry between domestic firms (assumed to be identical), and letting \(q_{1j} = q_1\) and \(d_{1j} = d_1\) in country 1, its military authority maximizes military capability

\[
M_1 = \left[ w_1 n_1 (q_1 d_1)^\alpha + (1 - w_1) \sum_{j=n_1+1}^{N} (u_{1j} m_{1j})^\alpha \right]^{\frac{1}{\alpha}}
\]

with respect to \(n_1 \geq 0, d_1\) and \(q_1\) given the world price \(P_j = \frac{c}{a}\) of variety \(j = n_1 + 1, n_1 + 2, \ldots, N\), the numbers of firms in the rest of the world, \(n_2, n_3, \ldots, n_\ell\) and two constraints. These are the budget constraint \((BC_1)\) and the representative domestic firm’s participation constraint \((PC_1)\) given by

\[
BC_1 : p_1 n_1 d_1 + \sum_{j=n_1+1}^{N} P_j m_{1j} = G_1
\]

\[
PC_1 : \pi_1 = (p_1 - c) d_1 + (P - c) x_1 - F - f q_1^\beta \geq 0
\]

where we have put \(P_1 = P_2 = \cdots = P_{n_1} = P\) in country 1. Clearly the PC constraint must bind so the procurement price is given by

\[
p_1 = c + \frac{F + f q_1^\beta - (P - c) x_1}{d_1} = c + \frac{H(q_1) - R(x_1)}{d_1}
\]

where we have written export revenue \((P - c) x_1 = R(x_1)\) and total fixed production costs as \(H(q_1) = F + f q_1^\beta\). It is convenient to eliminate the PC constraint by substituting for \(p_1\) in the \(BC_1\) constraint. This now becomes

\[
BC_1 : n_1 (cd_1 + H(q_1) - R(x_1)) + \sum_{j=n_1+1}^{N} P_j m_{1j} = G_1
\]

and the military authority now maximizes \(M_1\) given by (1) with respect to \(n_1, d_1\) and \(q_1\) given (25) \(P, n_2, n_3, \ldots, n_\ell\) and \(d_2, d_3, \ldots, d_\ell\). Notice that exports \(x_1\) depends only on \(d_2, d_3, \ldots, d_\ell, q_1\) and \(P_j\) (which we will confirm). To carry out this constrained optimization, define a Lagrangian

\[
L = M_1 - \lambda [n_1 (cd_1 + H(q_1) - R(x_1)) + \sum_{j=n_1+1}^{N} P_j m_{1j} - G_1]\]
The first-order conditions for an internal solution (where \(n_1 \geq 0\) is not binding) are then

\[
\begin{align*}
n_1 & : \frac{M_1^{1-\alpha}}{\alpha}[w_1 q_1^\alpha d_1^\alpha]q_1^\alpha - \lambda (cd_1 + H(q_1) - R(x_1)) = 0 \quad (27) \\
d_1 & : M_1^{1-\alpha}w_1 d_1^{\alpha-1} q_1^\alpha - \lambda c = 0 \quad (28) \\
m_{1j} & : M_1^{1-\alpha}(1-w_1)m_{1j}^{\alpha-1} u_{1j}^\alpha - \lambda P_j = 0; j = n_1 + 1, \ldots, N \quad (29) \\
q_1 & : M_1^{1-\alpha}q_1^{\alpha-1}w_1 d_1^\alpha - \lambda \left( \frac{\partial H}{\partial q_1} - \frac{\partial R}{\partial q_1} \right) = 0 \quad (30)
\end{align*}
\]

These 4 equations plus the constraint \(BC_1\) solve for \(n_1, d_1, m_{1j}, \lambda\) and \(q_1\). Dividing (27), (29) and (30) by (28), in turn, we can eliminate the shadow price \(\lambda\) to obtain

\[
\begin{align*}
d_1 & = \frac{\alpha[H(q_1) - R(x_1)]}{c(1-\alpha)} \quad (31) \\
m_{1j} & = d_1 \left( \frac{u_{1j}}{q_1} \right)^{\sigma-1} \left( \frac{c(1-w_1)}{P_j w_1} \right)^\sigma; j = n_1 + 1, \ldots, N \quad (32) \\
cd_1 & = \beta f q_1^\beta - (\sigma - 1)(P - c)x_1 \quad (33)
\end{align*}
\]

Notice that from (32), the first-order condition arising from the \(BC_1\) constraint, the relative demand for two imported goods with varieties \(j = r, s\) is given by

\[
\frac{m_{1r}}{m_{1s}} = \left( \frac{u_{1r}}{u_{1s}} \right)^{\sigma-1} \frac{P_s}{P_r} \quad (34)
\]

which, from (11) agrees with the decision taken at stage 3. Thus the anticipated optimal mix at stage 1 is actually implemented at stage 3, a condition for the subgame-perfectness of the equilibrium. In (32) the quality \(u_{1j}\) is that imposed by the country producing variety \(j\); We assume that imports from country \(k\) have quality \(\gamma_k q_k\) where \(\gamma_k \leq 1\) indicates the restriction of exports by country \(k\) to a technology below the best available. Then \(\frac{u_{1j}}{q_i} = \frac{\gamma_j q_j}{q_i}\). Define

\[
\phi_{1j} = \left( \frac{u_{1j}}{q_1} \right)^{\sigma-1} \left( \frac{c(1-w_1)}{P_j w_1} \right)^\sigma = \left( \frac{\gamma_j q_j}{q_i} \right)^{\sigma-1} \left( \frac{\alpha(1-w_1)}{w_1} \right)^\sigma \quad (35)
\]

using \(P_j = P = \frac{c}{\alpha}\). Then \(m_{1j} = \phi_{1j} d_1\). Similarly for country \(i\) imports of variety \(j\) are \(m_{ij} = \phi_{ij} d_i\) where

\[
\phi_{ij} = \left( \gamma_j q_j \right)^{\sigma-1} \left( \frac{\alpha(1-w_i)}{w_i} \right)^\sigma \quad (36)
\]

where \(u_{ij}\) is the quality allowed to country \(i\) by the producer of variety \(j\). Notice that since \(u_{ij} \leq q_{ij}\), \(\sigma > 1\) and \(w_i \geq \frac{1}{2}\) it follows that \(\phi_{ij} < \alpha < 1\). To complete the solution we note
that exports of country 1 of variety $j$ are
\[ x_1 = \sum_{i=2}^{\ell} m_{ij} = \sum_{i=2}^{\ell} \phi_{ij} d_i \] (37)

From (24) and (31) given the procurement price can now be written
\[ p_1 = c(1 + \frac{1 - \alpha}{\alpha}) = \frac{c}{\alpha} = P \] (38)
or, in other words, the procurement price is the world market price. The budget constraint
\[ n_1 = \frac{G_1 - PN m_1}{p_1 d_1 - P m_1} \]
completes the solution for the single economy given the decisions on $d_i$ and $q_i$ by the other countries.

### 3.3 The Symmetric Non-Cooperative Equilibrium

We now solve for a symmetric non-cooperative equilibrium in which $d_1 = d_2 = \cdots = d$, $\gamma_1 = \gamma_2 = \cdots = \gamma$, and other variables are defined similarly. Then $x_1 = x = (\ell - 1)\phi d$, $N = \ell n$. Substituting $R = (P - c)x$ we arrive at the symmetric equilibrium:

\[ d = \frac{\alpha \beta F}{c[\beta(1 - \alpha) - \alpha][1 + (\ell - 1)\phi]} \] (39)

\[ n = \frac{G}{P[1 + \phi(\ell - 1)]d} = \frac{G}{\beta F[\beta(1 - \alpha) - \alpha]} \] (40)

\[ q = \left[ \frac{c[1 + (\ell - 1)\phi]d}{\beta f} \right]^\frac{1}{2} = \left[ \frac{\alpha F}{f[\beta(1 - \alpha) - \alpha]} \right]^\frac{1}{2} \] (41)

\[ m = \phi d; \ x = (\ell - 1)m; \ p = P = \frac{c}{\alpha} \] (42)

It follows from (40) and (42) that the total output per firm or its ‘size’ is given by $y = d + x$ where
\[ y = \frac{\alpha \beta F}{c[\beta(1 - \alpha) - \alpha]} \] (43)

### 3.4 The Ramsey-Optimum and the Closed Economy

If countries coordinate in their choice of $n_i$ and $d_i$ and relax export controls, then they can reach the optimum (subject to a participation constraint for each firm), referred to in
the literature as the ‘Ramsey-optimum’ (RO)\textsuperscript{16}. If we continue to consider a symmetric outcome then the Ramsey-optimum is found by maximizing

\[
M = \left[ wn(qd)^\alpha + (1 - w)(N - n)(u\phi d)^\alpha \right]^{\frac{1}{\alpha}}
\]

(44)

where \( N = n\ell, \ u = q \) subject to the budget and participation constraints

\[
[p + P(\ell - 1)\phi]nd = G
\]

(45)

\[
(p - c)d + R(x) - H(q) = 0
\]

(46)

where as for the symmetric non-cooperative equilibrium above, \( R(x) = (P - c)x = (P - c)m = (P - c)\phi d \) and we recall the definition \( H(q) = F + fq^3 \). Thus we can rewrite (44) and (46) as

\[
M = \left[ (w + (1 - w)(\ell - 1)\phi^\alpha)n \right]^{\frac{2}{\alpha}}qd
\]

(47)

\[
[p + P(\ell - 1)\phi - c(1 + (\ell - 1)\phi)]d - H(q) = 0
\]

(48)

This optimization problem is equivalent to that of the closed economy with a procurement price \( p + P(\ell - 1)\phi \) and and marginal cost \( c(1 + (\ell - 1)\phi) \). Eliminating \( p + P(\ell - 1)\phi \) from (45) and \( d \) from (46) the problem reduces to the unconstrained maximization of \( M \) with respect to \( n \) and \( q \). It is straightforward to show that this leads to the following solution

\[
d^{RO} = \frac{\alpha F}{c[1 + (\ell - 1)\phi][\beta(1 - \alpha) - \alpha]} \]

(49)

\[
n^{RO} = \frac{G}{\beta F[\beta(1 - \alpha) - \alpha]} \]

(50)

\[
q^{RO} = \left[ \frac{c[1 + (\ell - 1)\phi]d^{RO}}{\beta f} \right]^{\frac{1}{\beta}} = \left[ \frac{\alpha F}{f[\beta(1 - \alpha) - \alpha]} \right]^{\frac{1}{\beta}}
\]

(51)

\[
m^{RO} = \phi d^{RO} \ ; \ x^{RO} = (\ell - 1)m^{RO} \ ; \ p^{RO} = P = \frac{c}{\alpha}
\]

(52)

Thus comparing these and previous results we immediately see that the Ramsey-optimum and the non-cooperative non-strategic equilibrium are the same. To summarise:

\textsuperscript{16}This is not the social optimum for two reasons: first, we cannot equate military capability with welfare. Second, military capability is optimal where price equals marginal cost. Then, to finance fixed costs, firms would need to be subsidised. This raises issues associated with distortionary taxes which lie outside the scope of this paper.
**Proposition 1**
The non-cooperative non-strategic equilibrium and the Ramsey-optimum coincide so there are no benefits from coordination in the choice of market structure and quality. In the closed-economy equilibrium firm size and number, and the quality of each variety is as for the non-cooperative equilibrium and Ramsey optimum.

### 3.5 Comparative Statics

We are interested in the effects of changes in the parameter

$$\phi = \gamma^{\sigma - 1} \left( \frac{\alpha(1-w)}{w} \right)^{\sigma}$$

through changes in the arms export parameter $\gamma \in (0, 1]$ and/or the preference parameter $w$. An increase in ‘openness’ is associated with a relaxation of arms export controls (a rise in $\gamma$) and/or a lower priority for domestic procurement (a fall in $w$) thus causing $\phi$ to rise. It is immediately apparent that $\frac{dd}{d\phi} < 0$ and $\frac{dm}{d\phi} > 0$. Otherwise the number of firms, the quality of output and the total size of the industry are independent of $\phi$ and therefore those factors affecting openness. To summarise:

**Proposition 2**

In a symmetric non-strategic equilibrium the only effect of an increase in ‘openness’ is to shift military expenditure from domestic to imported procurement. The number of firms, the size of the average firm and quality are independent of openness. The procurement price equals the world price.

The other parameters of interest are $G/F$, $F/c$, $F/f$, $\beta$ and $\alpha$. Clearly the number of firms, $n$, falls as $G/F$ falls, or $\alpha$ rises. Furthermore, differentiating (40) we have

$$\frac{\partial n}{\partial \beta} = \frac{\alpha G}{F\beta^2} > 0$$

and so $n$ falls as $\beta$ falls. From (41), as $\alpha$ rises and $\beta$ falls then the quality rises. The total size of the firm is given by (43) and is independent of $G$, rises as $\alpha$ rises and $\beta$ falls. We summarise these results in our final proposition:

**Proposition 3**

In both the non-cooperative equilibrium and the closed economy, the number of firms falls if total military expenditure $G$ falls, $F$ rises, military goods become more homogeneous ($\alpha \to 1$) and the quality cost parameter $\beta$ falls. The latter
three changes are associated with a rise in the size of the firm and a rise in quality, but changes in G do not affect firm size and quality.

These results for our stylized symmetric model reproduce the main empirical features of the defence in the defence industry set out in the previous section. The most obvious exogenous change to the industry is the fall in military expenditure G. In our procurement and trade equilibrium each government responds to such a change by concentrating production in fewer firms that remain of the same size. Quality is unaffected but military capability falls because product diversity falls. Globalisation in the form of a change in openness does not in itself affect market structure, but only brings about a switch between domestic and imported procurement. A rise in the parameters affecting the cost of quality, $\beta$ and $f/c$, not surprisingly leads to a drop in quality. Less obvious is the effect of changes in $\beta$ and the substitutability parameter $\alpha$ on quality, and firm size and number, as described in proposition 3. The basic intuition here is that quality and diversity are substitutes providing military capability in different ways. Faced with an decrease in $\beta$ the policymaker trades off a increase in quality with an decrease in diversity concentrating production in fewer production units of a greater size. Finally as $\alpha$ rises then goods become close substitutes and the benefits of diversity fall. The optimal response is then to reduce diversity and concentrate production in fewer units of a larger size. The fact that average size has not changed for either the US or the rest of the world implies that any changes to these various parameter that affect size were either small or that they tended to cancel each other out. Our calibration below suggests the latter: $\alpha$ seems to have fallen substantially, as has $\beta$. The first of these changes would decrease firm size, the second increases it, so the effects do seem to cancel.

Although the model is very abstract it allows us to analyse the relationship between the relevant set of variables: total demand, fixed costs, the effectiveness of R&D, competition and concentration. However it is important to be aware of what the model does not do. Firstly, in this model firms do not behave strategically, thus the increase in concentration and competition is not driven by the strategic behaviour of firms. This is quite unlike the case of telephone switches analysed in Sutton (1998, ch5). Normally in high R&D industries one would expect $\beta$ to be low: because the extra quality obtained per unit of R&D is high, it is profitable to invest in R&D. But this applies to R&D done by firms
to give them a competitive advantage over other firms. In the arms industry R&D is chosen by governments to give them a military advantage over their adversaries. Even if $\beta$ is large, so that the last 1% of performance enhancement is very expensive, that slight quality advantage may be worth having in combat. This makes defence a high R&D industry with high $\beta$.\(^{17}\) Secondly, the model does not explain the industrial dynamics associated with the rise in concentration, e.g. the process of mergers and acquisitions. We have assumed each firm produces a single variety. In practice, firms may produce more than one variety and there may be economies of scope.

### 3.6 Calibration of the Model

In this subsection we treat the US as an approximately closed economy and the rest of the world’s main exporters as a second bloc of economies open to each other, but closed to the rest of the world. The model generates a number of equally sized firms rather than the highly skewed distribution observed in practice. However we can calibrate the model using the inverse Herfindahl given in table 1, which can be given the interpretation of the number of equivalent firms in the market. This fell from 24 in 1990 to 9 in 1998 in the US and from 27 to 20 in the rest of the world. We can calibrate on these numbers of firms and the figures for total sales in table 1. Suppose we have $G$ falling from 115000 to 85000 in the US (both measured in millions of 1995 dollars), and from 78000 to 69000 in the rest of the world. We also use R&D data for the US which rose from 32% to 43% of total weapons procurement costs in the US over that period and for the UK (which we take as representative of the main producers in the rest of the world) which rose from 21% to 22%, SIPRI (2001)\(^{18}\).

We exploit the relationship giving the number of firms, (40). In addition from the binding participation constraint we have that revenue equals total costs, $P(d + x) = Py$ where we recall that $d =$domestic procurement, $x =$exports and $y = d + x =$output, all per firm. In equilibrium the procurement price equals the international market price $P = \frac{c}{\alpha}$ where $c =$marginal cost (equals average production cost given our assumption of constant

\(^{17}\)Kirkpatrick (1995) discusses this in more detail.

\(^{18}\)In 1998 the US spent 15% of total military expenditure, which includes other costs than procurement, on R&D, the UK 10%, France 9%, Germany 5%, thus the UK is rather higher than the other countries. However, this higher ratio may be more typical of the major weapons systems we are interested in.
returns to scale). Thus we have
\[ Py = \frac{c}{\alpha} y = \text{Total Costs}(TC) = F + fq^\beta + cy \tag{54} \]
where \( q \) is quality. In (54) let us associate the second quality component of total costs with R&D, the third with variable cost leaving F as fixed capital cost. Denote the shares of fixed, R&D and variable cost in total cost as \( \gamma_F, \gamma_R \) and \( \gamma_V \). Thus
\[ \frac{cy}{\alpha} = \frac{\text{variable costs}}{\text{total cost}} = \alpha = \gamma_V \tag{55} \]

For each bloc as a whole, in a symmetric equilibrium
\[ G = nPy = nTC \tag{56} \]
Hence from (56) and (40)
\[ 1 = \frac{TC}{\beta F} = [\beta(1 - \alpha) - \alpha] \tag{57} \]
Putting \( \alpha = \gamma_V \) (from (55)) and substituting into (57) then gives
\[ \beta = \frac{\gamma_V}{\gamma_R} \tag{58} \]

Since we have only have data on R&D, we must make an additional assumption: at the beginning of the period in question (1990) we assume that variable and fixed capital costs are divided in the usual proportions of labour and capital; i.e., 7 : 3. Then variable and fixed capital costs as proportions of total costs are given by
\[ \gamma_V = 0.7(1 - \gamma_R), \quad \gamma_F = 0.3(1 - \gamma_R) \tag{59} \]
To summarise, given observations \( n = \hat{n}, G = \hat{G} \) and R&D shares in total costs \( \hat{\gamma}_F, \hat{\gamma}_V \) and \( \hat{\gamma}_R \) respectively, our calibrated values for \( \alpha, \beta, \gamma_F \) and \( F \) at the beginning of the period in 1990 are given by
\[ \alpha = \hat{\gamma}_V = 0.7(1 - \gamma_R) \\
\beta = \frac{\hat{\gamma}_V}{\hat{\gamma}_R} \\
\hat{\gamma}_F = 0.3(1 - \gamma_R) \\
F = \frac{\hat{G}}{\hat{n}} [\beta(1 - \alpha) - \alpha] \tag{60} \]

\footnote{We need to check the second order condition \( \beta + 1 > \sigma = \frac{1}{1 - \alpha} \) holds. Substituting from (55) and (58), the condition becomes \( \frac{\gamma_V}{\gamma_R} > \frac{1}{1 - \gamma_V} - 1 = \frac{\gamma_V}{1 - \gamma_V} \); i.e., \( \gamma_R < 1 - \gamma_V \). But \( \gamma_R = 1 - \gamma_V - \gamma_F < 1 - \gamma_V \), so the second order condition is satisfied with this calibration.}
For the US in 1990 this gives us calibrated values \( \alpha = \gamma_V = 0.476 \), \( \beta = 1.49 \) and \( \gamma_F = 0.204 \). For the rest of the world the corresponding values are: \( \alpha = \gamma_V = 0.553 \), \( \beta = 2.63 \) and \( \gamma_F = 0.237 \). These estimates suggest that the US could increase quality at less cost than the rest of the world. This is not implausible if there are learning curves or increasing returns to scale in R&D.

Can our model now provide a plausible explanation for the fall in firms numbers over the period? From (56) with \( P = \frac{\xi}{\alpha} \) we have that

\[
\frac{n_{98}}{n_{90}} = \frac{G_{98}\alpha_{98}c_{98}y_{98}}{G_{90}\alpha_{90}c_{98}y_{98}}
\]

Thus we can explain the fall in numbers by some combination of a fall in \( \alpha = \gamma_V \), total spending, and an increase in marginal cost and size. Our empirical analysis of concentration and the growth of firms suggest there has been no systematic increase in the size of firms (i.e., \( y_{90} = y_{98} \)). If we rule out an increase in marginal cost (i.e., \( c_{90} = c_{98} \)) then this leaves only two explanations: \( G \) has fallen (as observed) and fixed plus R&D costs as a proportion of total costs has increased (as partly observed). The implied changes over the period in variable and fixed costs as a proportion of total costs are summarised as:

<table>
<thead>
<tr>
<th>Country</th>
<th>Implied Change in ( \alpha = \gamma_V )</th>
<th>Implied Change in ( \gamma_F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.48 (1990); 0.24 (1998)</td>
<td>0.20(1990) 0.33(1998)</td>
</tr>
<tr>
<td>RoW</td>
<td>0.55 (1990); 0.44 (1998)</td>
<td>0.24(1990) 0.34 (1998)</td>
</tr>
</tbody>
</table>

This would explain the reduction in the number of firms in terms of a drop in demand, \( G \), and a shift in total costs from variable to fixed, in both the US and the rest of the world. However it is probable that the industry, particularly in the rest of the world had not yet reached the equilibrium concentration in 1998, as we have assumed here. There is also an irreversibility that leads to asymmetry, which we have not modelled. If a country, chooses to import and loses the capability to produce a particular major weapon system, then it is very expensive to re-acquire the capability. Increases in concentration can become irreversible. It seems unlikely that the US defence budget increases announced in 2002 will reverse the growth in concentration.
4 Conclusions

With the end of the Cold War the international arms industry was confronted by a massive reduction in demand for its products. This was also happening at a time when technology and government attitudes towards domestic production and ownership were already leading to changes in the supply side. This led to an increase in R&D as a proportion of total production costs within the companies, as companies responded to technological imperatives and contracted out component production to reduce costs. Analysing the data on the major arms producers shows clearly that at the end of the Cold War, the international arms industry was relatively unconcentrated by comparison with comparable high technology industries like commercial aerospace or pharmaceuticals. In fact it was quite close to the Sutton lower bound, a clear legacy of government’s historical support for domestic arms production in the major powers. It is, therefore, no surprise that concentration increased markedly 1990-98, but what is interesting is that this was not associated with increases in the average size of firms and that there should be no evidence of any tendency for large firms to grow faster.

In an attempt to determine the fundamentals of this process a trade model with optimal procurement decisions was constructed. This model predicted that concentration will increase with a decline in the total size of the market, increased fixed capital costs, and increased R&D costs and be associated with fewer firms of the same size. These are all characteristic of the post Cold War period. Concentration did increase with the five firm concentration ratio raising from just over 20% to over 40%. However, most of the concentration happened in the largest market, the US. Our analysis points to two opposite effects of increased concentration on competition. On the one hand the willingness of governments to procure from abroad means that although this does not alter world concentration (see proposition 2), firms no longer sell in a sheltered domestic market and competition increases. However this pro-competitive effect is offset by an decrease in the elasticity of substitution between varieties. This in turns increases the market power of each firm producing its own differentiated product.20 Despite the turmoil, the industry

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20This conclusion and proposition 2 in particular needs to be treated with caution for two reasons. First, we ignore strategic pricing in the Bertrand equilibrium. This becomes increasingly important as concentration increases. Second, in our standard Dixit-Stiglitz CES military capability function, taste for
is still not very concentrated by comparison with other comparable industries and it is probable that concentration has not yet reached its equilibrium level, particularly outside the US. This may raise political problems for countries concerned to protect their defence industrial bases. It is also possible that because of asymmetries the increases in military expenditure, which have recently been announced by some countries, will not reverse the trend to increased concentration.

Overall, the paper has provided a detailed understanding of the changes that have been taking place in the post Cold War arms industry. It has also provided a model of government behaviour which drives this market and goes some way to explaining the most important factors determining the changing structure of the international arms industry. The model provides a valuable starting point, but leaves a number of unanswered questions. These include explaining the skewed distribution of firm sizes and the firms choices about mergers and acquisitions, when permitted by the government, are not explained. Future research needs to consider the firm dynamics, develop the theoretical work on market structure and procurement and the influence of technological change on the military capability function. These are all subjects under investigation.

Data Appendix

The Stockholm International Peace Research Institute (SIPRI) have collected information on arms sales, total sales, profits and employment for the 100 largest arms-producing companies since 1988 and published it annually in their yearbook. We use 1990 as our starting year. Having data on total sales is a useful control, since arms producers tend to be the defence divisions of diversified firms. SIPRI send questionnaires to companies asking them for the information. In the case of the share of arms in total sales, companies may be unwilling to disclose this and in such cases SIPRI uses estimates constructed with the assistance of a network of country experts. Where a company enters the top 100, they will go back and try to collect data for earlier years, so we have some observations on

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variety is linked one-for-one with the elasticity of substitution. See Benassy (1996) for a discussion of this and a more general production function which disentangles these effects. Dunne et al (2002), generalises the model of this paper using this form of CES production function and including strategic effects. Then the the optimal choice of concentration by countries acting independently is below the Ramsey optimal, and increasing openness encourages this beggar-thy-neighbour choice of firm number, providing a further factor to explain the increase in world concentration.
companies ranked below 100. The definition of arms is not straightforward and does not match sales to defence ministries, the other main source of data. For instance, defence ministries spend large parts of their budgets on fuel and food, which would not count as arms. There may also be elements of double counting since some of the sales are of components or munitions to other companies. This is not a problem for the measurement of concentration but is a problem in using the total sales of these companies as a measure of the volume of arms supplied.

Looking at the top 100 makes the sample endogenous and there can be missing data for companies who are in the top 100 for a couple of years, dropping out and then re-entering. The measures of concentration in each year are probably reasonably accurate, because most of the inaccuracies are for the smaller firms and this will probably not influence concentration measures very much. The major problems arise for any dynamic analysis, since this requires some treatment mergers and acquisitions, which are a central feature of this industry. Consider the case of Lockheed-Martin, the product of the merger of Lockheed and Martin Marietta. This could be treated by assuming Lockheed continued Martin-Marietta exited; Lockheed exited Martin-Marietta continued; both exited and a new firm Lockheed-Martin entered. Analysis of entry-exit and Gibrat regressions will be sensitive to this treatment. Name change is not conclusive. Thales and BAE Systems, despite the name changes are clearly continuations of Tompson-CSF and British Aerospace. Using standard data, it might be reasonable to assume that the larger firm acquired the smaller firm, even though the reverse does happen. In the case of our data, this is not straightforward since it is not clear whether we should judge size by total sales or arms sales. For instance, should EADS should be treated as a continuation of Aerospatiale (arms sales $3,300m total sales $13,743) or Daimler Chrysler (arms sales $3,040 total sales $160,000m)? Having a large new company suddenly appear in the data, makes entry look too easy, but providing a history by allocating the merged company to a predecessor will inevitably be arbitrary. Similar problems arise when a company spins off one of its defence divisions.
References


