Unambiguous inference in sign-restricted VAR models

Robert Calvert Jump
University of the West of England, Bristol

Economics Working Paper Series
1802

Unambiguous inference in sign-restricted VAR models*

Robert Calvert Jump†

January 9, 2018

Abstract

This paper demonstrates how sign restrictions can be used to infer the signs of certain historical shocks from reduced form VAR residuals. This is achieved without recourse to non-sign information. The method is illustrated by an application to the AD-AS model using UK data.

Keywords: Structural VAR, sign restrictions.

JEL Codes: C51, C52.

*I would like to thank Yannis Dafermos, Adrian Pagan, Ron Smith, Harald Uhlig, and Don Webber for comments and advice. All errors are the responsibility of the author.

†Department of Accounting, Economics and Finance, University of the West of England, Coldharbour Lane, Bristol, BS16 1QY, UK. Email: rob.calvertjump@uwe.ac.uk.

1 Introduction

Sign restrictions are a well known method for identifying structural VAR models (Uhlig 2005, Fry and Pagan 2011). However, the method results in set identification rather than point identification (Lütkepohl and Kilian 2017). In practice, unless non-sign information such as a Bayesian prior or loss function is appealed to, sign restrictions generally yield a large set of impulse response functions, variance decompositions, or historical decompositions that are consistent with the data. It is therefore unsurprising that non-sign information is regularly utilised in the literature on sign-restricted VAR models.

Unfortunately, the choice of non-sign information will generally affect the conclusions drawn from a sign-restricted VAR model. This problem is well known; Lütkepohl and Kilian (2017) discuss it in the context of Uhlig's (2005) model. Given this, it is important to know if any inference from sign-restricted VAR models can be made unambiguously - i.e. without recourse to non-sign information. The present paper answers this question in the affirmative. Specifically, it demonstrates that sign restrictions can be used to infer the signs of certain historical shocks from reduced form VAR residuals without the use of non-sign information. The method is illustrated by an application to the AD-AS model using UK data.

2 Sign restrictions in VAR models

Consider a structural VAR model,

$$Az_t = \sum_{i=1}^{p} A_i z_{t-i} + \epsilon_t, \tag{1}$$

where z_t is a vector of length n, A and A_i are $n \times n$ parameter matrices, and ϵ_t is a vector white noise process with mean zero and variance-covariance matrix Ω_ϵ. Any deterministic terms are suppressed without loss in generality. From (1) it follows that,

$$z_t - \mathbb{E}[z_t | z_{t-1}, ..., z_{t-p}] = A^{-1}\epsilon_t = v_t, \tag{2}$$

where v_t are the innovations of the corresponding reduced form VAR. Write (2) as,

$$\begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \vdots \\ \epsilon_{it} \\ \vdots \\ \epsilon_{nt} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nj} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} v_{1t} \\ v_{2t} \\ \vdots \\ v_{it} \\ \vdots \\ v_{nt} \end{bmatrix}, \tag{3}$$

where a_{ij} are the elements of A. As is well known, the estimated variance-covariance matrix of v_t does not contain sufficient information to uniquely identify each a_{ij}.

To place some structure on the problem, suppose that we postulate a set of a priori sign restrictions on A. Specifically, suppose that for each a_{ij} in A we postulate $a_{ij} > 0$ or $a_{ij} < 0$. From (3) we then have,

$$\begin{aligned} a_{ij}v_{jt} > 0 \ \forall \ j = 1, ..., n \ \Rightarrow \ \epsilon_{it} > 0, \\ a_{ij}v_{jt} < 0 \ \forall \ j = 1, ..., n \ \Rightarrow \ \epsilon_{it} < 0, \\ \text{otherwise} \ \ \epsilon_{it} \lesseqgtr 0. \end{aligned} \tag{4}$$

Note that the event $v_{jt} = 0$ is not considered in (4), as most reasonable innovation distributions imply a zero probability of this event occurring. The fact that we can only infer (at most) the sign of ϵ_{it} in (4) is due to the fact that we only impose sign restrictions on the elements of A, and we do not appeal to non-sign information to pin down their magnitudes from the estimated reduced form VAR.

It is therefore possible to combine a series of reduced form residuals with sign restrictions on A to infer the signs of certain structural shocks, using (4). To make the evaluation of the conditions in (4) more straightforward, consider an $n \times n$ matrix B where $b_{ij} = 1$ if $a_{ij} > 0$ and $b_{ij} = -1$ if $a_{ij} < 0$, i.e. $B = \text{sgn}(A)$. Similarly, consider a vector ξ_t, where $\xi_{it} = 1$ if $v_{it} > 0$ and $\xi_{it} = -1$ if $v_{it} < 0$, i.e. $\xi_t = \text{sgn}(v_t)$. Both B and ξ_t are straightforward to code in econometrics software packages given the set of sign restrictions and reduced form residuals from an estimated VAR model. Denoting the ith row of B as b_{i*}, from (3) and (4) it follows that,

$$\begin{aligned} b_{i*}\xi_t = n \ \Rightarrow \ \epsilon_{it} > 0, \\ b_{i*}\xi_t = -n \ \Rightarrow \ \epsilon_{it} < 0, \\ \text{otherwise} \ \ \epsilon_{it} \lesseqgtr 0. \end{aligned} \tag{5}$$

Note that b_{i*} is a row vector and ξ_t is a column vector, so $b_{i*}\xi_t = \sum_{j=1}^{n} b_{ij}\xi_{jt}$.

The inference resulting from (4) or (5) unambiguously follows from the reduced form VAR estimates combined with sign restrictions on A, with no recourse to non-sign information. Incidentally, it also requires no assumptions over the variance-covariance matrix Ω_ϵ of the structural shocks.

3 Example: A bivariate AD-AS model

As an example of the method outlined in section 2, suppose the endogenous variables in the VAR in (1) are $z_t = (u_t, \pi_t)$, where u_t is the unemployment rate and π_t is the inflation rate. The model can be thought of as an AD-AS model, with,

$$\begin{bmatrix} \epsilon_{Dt} \\ \epsilon_{St} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_{ut} \\ v_{\pi t} \end{bmatrix}, \tag{6}$$

where ϵ_{Dt} is an aggregate demand shock, ϵ_{St} is an aggregate supply shock, v_{ut} is the reduced form unemployment innovation, and $v_{\pi t}$ is the reduced form inflation innovation. Reasonable sign restrictions are $a_{11} < 0$, $a_{12} > 0$, $a_{21} < 0$, and $a_{22} < 0$. As a result, a positive aggregate demand shock will decrease unemployment and increase inflation, and a positive aggregate supply shock will decrease unemployment and decrease inflation.

There are four possible patterns of reduced form innovation signs, giving rise to four possible ξ_t: $\xi_t = [1, 1]'$, $\xi_t = [-1, -1]'$, $\xi_t = [1, -1]'$, and $\xi_t = [-1, 1]'$. There are then four possible $B\xi_t$ that can occur in any given period, allowing us to infer the following structural shock signs using (5):

1. $B\xi_t = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \Rightarrow \begin{matrix} \epsilon_{Dt} \lesseqgtr 0 \\ \epsilon_{St} < 0 \end{matrix},$

2. $B\xi_t = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \Rightarrow \begin{matrix} \epsilon_{Dt} \lesseqgtr 0 \\ \epsilon_{St} > 0 \end{matrix},$

3. $B\xi_t = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} \epsilon_{Dt} < 0 \\ \epsilon_{St} \lesseqgtr 0 \end{matrix},$

4. $B\xi_t = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} \epsilon_{Dt} > 0 \\ \epsilon_{St} \lesseqgtr 0 \end{matrix}.$

So, in case 1, positive unemployment and inflation innovations at time t imply the existence of a negative aggregate supply shock at time t, but provide no inference about the sign of the aggregate demand shock. In this simple example, we can infer the sign of a single structural shock in each period.

To illustrate the method, a reduced form VAR model in the unemployment rate and inflation rate is estimated using the Bank of England's "millennium of macroeconomic data" dataset[1]. The data is annual and the sample period is 1900 - 2016, which allows a number of important sources of shocks to be contained within the sample. The Akaike information criterion suggests a three lag model. The resulting VAR does not suffer from autocorrelation at the 5% level for lags 1-4, and the parameter estimates appear stable, as illustrated by CUSUM plots and recursive parameter estimates available upon request.

The reduced form residuals are plotted in figure 1, from which the large forecast errors during the war periods and 1920s are immediately apparent. Table 1 presents unemployment and inflation residuals and the implied demand and supply shocks from the reduced form VAR, using the sample of residuals plotted in figure 1 and the method outlined above. To reduce the size of the table, only those years for which at least one of the unemployment and

[1]Specifically, this is the a-millennium-of-macroeconomic-data-for-the-uk.xlsx sheet, downloaded on 02/01/2018 from www.bankofengland.co.uk/research/Pages/datasets/default.aspx. The LFS consistent measure of the unemployment rate and the consumer price inflation series are used, both from tab "A1. Headline Series".

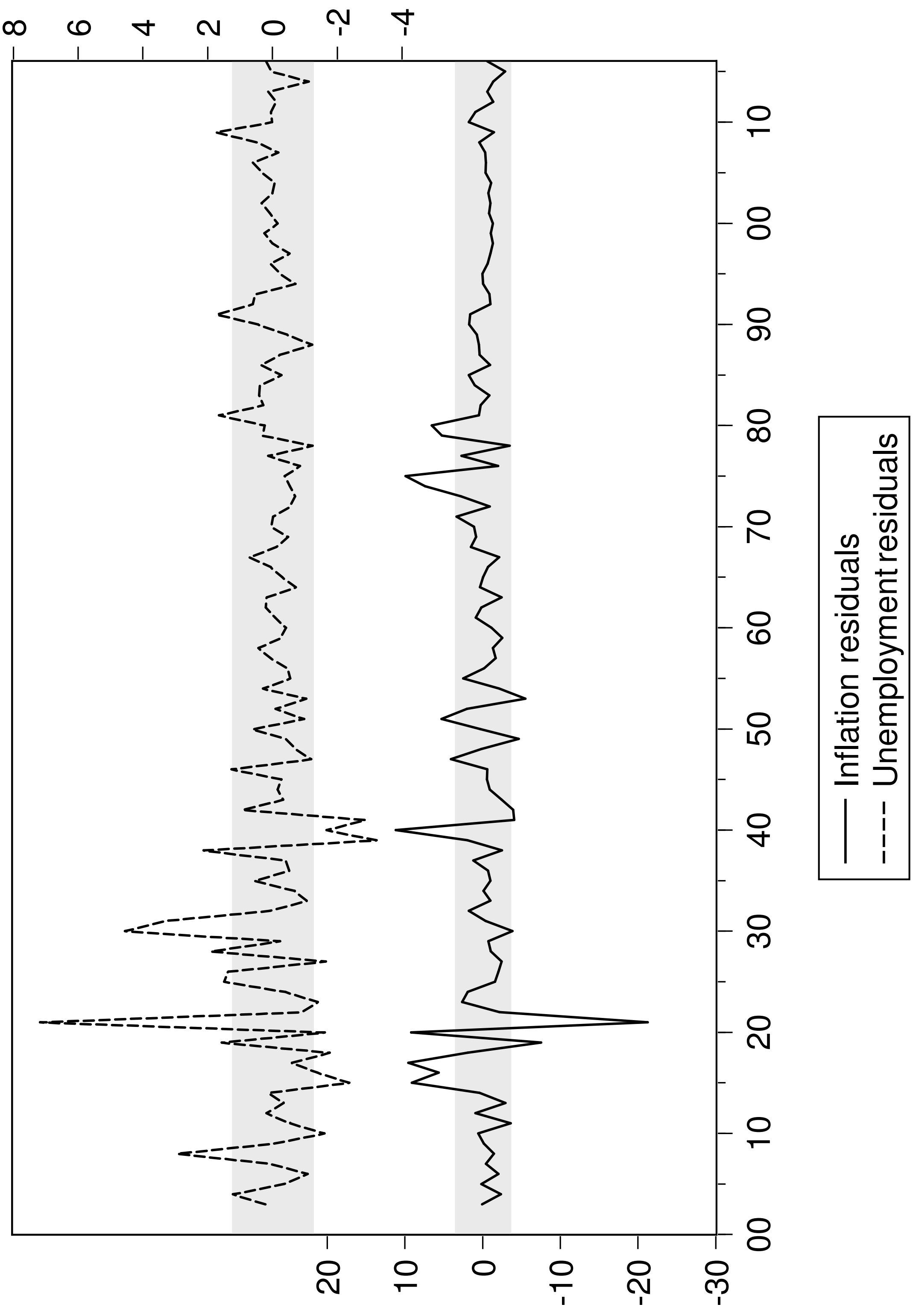

Figure 1: VAR residuals, 1903 - 2016, with ±1 standard deviation regions shaded in grey.

Date	$\hat{v}_t^u$	$\hat{v}_t^\pi$	Implied demand shock	Implied supply shock
1908	2.94	-1.45	−	
1910	-1.61	0.58	+	
1915	-2.38	9.09	+	
1916	-1.39	5.63	+	
1917	-0.61	9.55	+	
1918	-1.76	1.92	+	
1919	1.57	-7.54	−	
1920	-1.61	9.20	+	
1921	7.17	-21.22	−	
1923	-1.40	2.63	+	
1925	1.49	-1.59	−	
1926	1.36	-2.06	−	
1927	-1.64	-2.44		+
1928	1.90	-1.04	−	
1930	4.55	-3.82	−	
1931	3.35	-0.36	−	
1938	2.13	-2.48	−	
1939	-3.21	1.95	+	
1940	-1.67	11.15	+	
1941	-2.86	-4.05		+
1942	0.93	-3.95	−	
1947	-1.20	4.08	+	
1949	-0.41	-4.64		+
1951	-0.98	5.29	+	
1953	-1.046	-5.52		+
1974	-0.54	7.39	+	
1975	-0.38	9.90	+	
1979	0.30	5.26		−
1980	0.24	6.53		−
1981	1.65	0.46		−
1991	1.73	1.61		−
2009	1.74	-1.44	−	

Table 1: Implied demand and supply shocks. Only those years in which at least one of $|v_t^u|$ and $|v_t^\pi|$ are greater than their respective sample standard deviations are displayed.

inflation residuals are greater in magnitude than their respective sample standard deviations are presented. On examination, the table indicates a broad consistency of the AD-AS model with the UK historical record. Positive demand shocks are present in the war periods, and negative demand shocks are present during the Great Depression and the Great Recession. Negative supply shocks are present following the 1979 oil price shock, and there are no reported shocks during the two periods of post-war tranquillity, from 1954 - 1973 and 1992 - 2008.

4 Summary

Non-sign information is regularly appealed to in the literature on sign-restricted VAR models. This is understandable as, without Bayesian priors, loss functions, or similar, inference is often highly ambiguous. Unfortunately, the choice of non-sign information will generally affect the conclusions, and it is therefore important to know if any inference from sign-restricted VAR models can be made without recourse to non-sign information. The present paper answers this question in the affirmative.

The method proposed here could be extended in a number of ways. In particular, bounds on the size of shocks could be computed without recourse to non-sign information, at least for small models. This would permit an approximation to a full historical decomposition, and awaits future work.

References

[1] Fry, R., and Pagan, A. 2011. Sign restrictions in structural vector autoregressions: A critical review. *Journal of Economic Literature*, 49.4, 938-960.

[2] Lütkepohl, H., and Kilian, L. 2017. *Structural Vector Autoregressive Analysis*, Cambridge: Cambridge University Press (forthcoming).

[3] Uhlig, H. 2005. What are the effects of monetary policy on output? Results from an agnostic identification procedure. *Journal of Monetary Economics*, 52.2, 381-419.

Recent UWE Economics Papers
See http://www1.uwe.ac.uk/bl/research/bcef/publications.aspx for a full list.

2018

1802 **Unambiguous inference in sign-restricted VAR models**
Robert Calvert Jump

1801 **Degree algorithms, grade inflation and equity: the UK higher education sector**
David O. Allen

2017

1706 **Internal rationality, heterogeneity and complexity in the new Keynesian model**
Cars Hommes, Robert Calvert Jump and Paul Levine

1705 **The regionalization of national input–output tables: a study of South Korean regions**
Anthony T. Flegg and Timo Tohmo

1704 **The impact of quantitative easing on aggregate mutual fund flows in the UK**
Iris Biefang-Frisancho Mariscal

1703 **Where are the female CFOs?**
Gail Webber, Don J Webber, Dominic Page and Tim Hinks

1702 **Mental health and employment transitions: a slippery slope**
Don J Webber, Dominic Page and Michail Veliziotis

1701 **SMEs access to formal finance in post-communist economies: do institutional structure and political connectedness matter?**
Kobil Ruziev and Don J Webber

2016

1611 **Curriculum reform in UK economics: a critique**
Andrew Mearman, Sebastian Berger and Danielle Guizzo

1610 **Can indeterminacy and self-fulfilling expectations help explain international business cycles?**
Stephen McKnight and Laura Povoledo

1609 **Pricing behaviour and the role of trade openness in the transmission of monetary shocks**
Laura Povoledo

1608 M**easuring compliance with minimum wages**
Felix Ritchie, Michail Veliziotis, Hilary Drew and Damian Whittard

1607 **Can a change in attitudes improve effective access to administrative data for research?**
Felix Ritchie

1606 **Application of ethical concerns for the natural environment into business design: a novel business model framework**
Peter Bradley, Glenn Parry and Nicholas O'Regan

1605 **Refining the application of the FLQ Formula for estimating regional input coefficients: an empirical study for South Korean regions**
Anthony T. Flegg and Timo Tohmo

1604 **Higher education in Uzbekistan: reforms and the changing landscape since independence**
Kobil Ruziev and Davron Rustamov

1603 **Circular economy**
Peter Bradley

1602 **Do shadow banks create money? 'Financialisation' and the monetary circuit**
Jo Michell

1601 **Five Safes: designing data access for research**
Tanvi Desai, Felix Ritchie and Richard Welpton

2015

1509 **Debt cycles, instability and fiscal rules: a Godley-Minsky model**
Yannis Dafermos

1508 **Evaluating the FLQ and AFLQ formulae for estimating regional input coefficients: empirical evidence for the province of Córdoba, Argentina**
Anthony T. Flegg, Leonardo J. Mastronardi and Carlos A. Romero

1507 **Effects of preferential trade agreements in the presence of zero trade flows: the cases of China and India**
Rahul Sen, Sadhana Srivastava and Don J Webber

1506 **Using CHARM to adjust for cross-hauling: the case of the Province of Hubei, China**
Anthony T. Flegg, Yongming Huang and Timo Tohmo

1505 **University entrepreneurship education experiences: enhancing the entrepreneurial ecosystems in a UK city-region**
Fumi Kitagawa, Don J. Webber, Anthony Plumridge and Susan Robertson

1504 **Can indeterminacy and self-fulfilling expectations help explain international business cycles?**
Stephen McKnight and Laura Povoledo