Empirical Model Discovery and Theory Evaluation

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‘Any sufficiently advanced technology is indistinguishable from magic.’ Arthur C. Clarke, Profiles of The Future, 1961
Economic theory main basis for econometric models, but: many features of models not derivable from theory.

Need empirical evidence on which:

[a]: variables are actually relevant (specification),
[b]: their lagged responses (dynamic reactions),
[c]: functional forms of relationships (non-linearities),
[d]: structural breaks & unit roots (non-stationarities),
[e]: simultaneity (or exogeneity), expectations, etc.

Almost always must be data-based on available sample: need to discover what matters empirically.

Theory provides an object for modelling— but:
(A) embed that object in much more general formulation;
(B) search for the simplest acceptable representation;
(C) evaluate the findings.

How to accomplish? And what are its properties?
Data generation process (DGP): joint density of all variables in economy

Impossible to accurately theorize about or model precisely Too high dimensional and far too non-stationary.

Need to reduce to manageable size in ‘local DGP’ (LDGP): the DGP in space of $n$ variables $\{x_t\}$ being modelled

Theory of reduction explains derivation of LDGP: joint density $D_x(x_1 \ldots x_T | \theta)$. Acts as DGP, but ‘parameter’ $\theta$ may be time varying

Knowing LDGP, can generate ‘look alike data’ for $\{x_t\}$ which only deviate from actual data by unpredictable noise

Once $\{x_t\}$ chosen, cannot do better than know $D_x(\cdot)$—so the LDGP $D_x(\cdot)$ is the target for model selection: need to relate theory model to that target.
Choice of \( n \) variables, \( \{x_t\} \), to analyze is fundamental: determines the modelling target LDGP, \( D_x(\cdot) \), and its properties.

Prior reasoning, theoretical analysis, previous evidence, historical and institutional knowledge all important.

**Should be 90 + % of effort in an empirical analysis.**

Aim to avoid complicated and non-constant LDGPs. Crucial not to omit substantively important variables: small set \( \{x_t\} \) more likely to do so.

Given \( \{x_t\} \), have defined the target \( D_x(\cdot) \).

Now embed that target in a general model formulation, which also retains, **but does not impose**, the theory-based variables.
Sample of $T$ observations, $\{x_t\} = \{y_t, z_t\}$: but no theory specification of unit of time, observations may be contaminated (measurement errors), underlying processes integrated, abrupt unanticipated shifts induce various forms of breaks. All these aspects must be discovered empirically: model selection is inevitable and ubiquitous. So how to utilize economic analyses efficiently if cannot impose theory empirically?

(1) **Discovery in general**

(2) Automatic model extension

(3) Automatic model selection

(4) Automatic model evaluation

(5) Embedding theory models

(6) Excess numbers of variables $N > T$

(7) Non-invariance of NKPCs

**Conclusions**
Discoveries in economics mainly from theory. But all economic theories are: (a) incomplete; (b) incorrect; and (c) mutable.

(a) Need strong *ceteris paribus* assumptions: *inappropriate in a non-stationary, evolving world.*

(b) Consider an economic analysis which suggests:

\[ y = f(z) \]  

where \(y\) depend on \(n\) ‘explanatory’ variables \(z\). Form of \(f(\cdot)\) in (1) depends on:

utility or loss functions of agents, constraints they face, & information they possess.

Analyses arbitrarily assume: forms for \(f(\cdot)\), that \(f(\cdot)\) is constant, that only \(z\) matters, & that the \(z\)s are ‘exogenous’.

Yet must aggregate across heterogeneous individuals whose endowments shift over time, often abruptly.
(c) Economic analyses have changed the world, and our understanding: from the ‘invisible hand’ in Adam Smith’s *Theory of Moral Sentiments* (1759, p.350) onwards, theory has progressed dramatically—key insights into option pricing, auctions and contracts, principal-agent and game theories, trust and moral hazard, asymmetric information, institutions: major impacts on market functioning, industrial, and even political, organization.

But imagine imposing 1900’s economic theory in empirical research today.

Much past applied econometrics research is forgotten: discard the economic theory that it ‘quantified’ and you discard the associated empirical evidence.

Hence fads & fashions, ‘cycles’ and ‘schools’ in economics.
Discovery: learning something previously unknown. Cannot know how to discover what is not known—unlikely there is a ‘best’ way of doing so.

Many empirical discoveries have element of chance:
- luck: Fleming—penicillin from a dirty petrie dish
- serendipity: Becquerel—discovery of radioactivity
- ‘natural experiment’: Dicke—role of gluten in celiac disease
- trial and error: Edison—incandescent lamp
- brilliant intuition: Faraday—dynamo from electric shock
- false theories: Kepler—regular solids for planetary laws
- valid theories: Pasteur—germs not spontaneous generation
- systematic exploration: Lavoisier—oxygen not phlogiston
- careful observation: Harvey—circulation of blood
- new instruments: Galileo—moons around Jupiter
- self testing: Marshall—ulcers caused by *Helicobacter pylori*. 
Theoretical discoveries also important.

Classic examples include:

- uniform motion: Galileo Galilei;
- universal gravitation: Issac Newton;
- electro-magnetic spectrum: Clerk Maxwell;
- black-body radiation: Max Planck;
- relativity: Albert Einstein;
- quantum theory: Niels Bohr;
- positron: Paul Dirac;
- quark: Murray Gell-Mann.

Some ‘evidence based’; some ‘thought experiments’. All required later independent evaluation.
Science is both inductive and deductive. Must distinguish between:
context of discovery—where ‘anything goes’, and
context of evaluation—rigorous attempts to refute.

However a discovery made, needs a warrant that it is ‘real’. Methods of evaluation are subject-specific: economics requires a theoretical interpretation consistent with ‘mainstream theory’.

Accumulation and consolidation of evidence crucial: data reduction a key attribute of science (think $E = mc^2$).
Seven aspects in common to above examples of discovery. First, \textit{theoretical context}, or framework of ideas. Second, going \textit{outside} existing state of knowledge. Third, \textit{searching} for something. Fourth, \textit{recognition} of significance of what is found. Fifth, \textit{quantification} of what is found. Sixth, \textit{evaluating} discovery to ascertain its ‘reality’. Seven, \textit{parsimoniously summarize} information acquired. But science perforce is simple to general–a slow and uncertain route to new knowledge. Econometrics discovery need not be....
Classical econometrics: covert discovery

Postulate:

\[ y_t = \beta' z_t + \epsilon_t, \quad t = 1, \ldots, T \]  \hspace{1cm} (2)

Aim to obtain ‘best’ estimate of the constant parameters \( \beta \), given the \( n \) correct variables, \( z \), ‘independent’ of \( \{ \epsilon_t \} \) and uncontaminated observations, \( T \), with \( \epsilon_t \sim \text{IID}[0, \sigma^2_\epsilon] \).

Many tests to ‘discover’ departures from assumptions of (2), followed by recipes for ‘fixing’ them—covert and unstructured empirical model discovery.

Model selection: discovering the ‘best’ model.

Starts from (2) assuming \( N \) ‘correct’ initial \( z \), accurate data over \( T \), constant \( \beta \) and valid conditioning.

Aim to ‘discover’ the subset of relevant variables, \( z^*_t \).

Selected ‘best model’ may be poor approximation to LDGP: almost never evaluated.
Robust statistics: discovering the best sample

Same start (2), but aim to find a ‘robust’ estimate of a constant $\beta$ by selecting over $T$.

Worry about data contamination and outliers, so select sample, $T^*$, where outliers least in evidence, given correct set of relevant variables $z$.

All other difficulties still need separate tests, and must be fixed if found.

$z$ rarely selected jointly with $T^*$, so assumes $z = z^*$.

Similarly for non-parametric methods:

aim to discover ‘best’ functional form or distribution, assuming correct $z$, no data contamination, constant $\beta$, etc., all rarely checked.

Each assumes away what the others tackle.
Need to tackle them all jointly.
Re-frame empirical modelling as discovery process: part of a progressive research strategy.

Starting from $T$ observations on $N > n$ variables $z$, aim to find $\beta^*$ for $s$ lagged functions $g(z^*_t) \ldots g(z^*_{t-s})$ of a subset of $n$ variables $z^*$, jointly with $T^*$ and $\{1\{t=t_i\}\}$—indicators for breaks, outliers etc.

Embeds initial economic analysis $y = f(z)$, but in a much more general initial model.

Globally, learning must be simple to general; but locally, need not be.

General approach explained in Castle, Doornik and Hendry (2010).
Implications for automatic methods

Same seven stages as for discovery in general.

First, theoretical derivation of the relevant set $x$.

Second, going outside current view by automatic creation of a general model from $x$ embedding $y = f(z)$.

Third, search by automatic selection to find viable representations: too large for manual labor.

Fourth, criteria to recognize when search is completed: congruent parsimonious-encompassing model.

Fifth, quantification of the outcome: translated into unbiasedly estimating the resulting model.

Sixth, evaluate discovery to check its ‘reality: new data, new tests or new procedures. Can also evaluate the selection process itself.

Seventh, summarize vast information set in parsimonious but undominated model.
(1) Discovery in general
(2) **Automatic model extension**
(3) Automatic model selection
(4) Automatic model evaluation
(5) Embedding theory models
(6) Excess numbers of variables $N > T$
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**Conclusions**
Extensions determine how well LDGP is approximated

Create three extensions automatically:

(i) lag formulation to implement sequential factorization;
(ii) functional form transformations for non-linearity;
(iii) impulse-indicator saturation (IIS) for parameter non-constancy and data contamination.

(i) Create \( s \) lags \( x_t \ldots x_{t-s} \) to formulate general linear model:

\[
y_t = \beta_0 + \sum_{i=1}^{s} \lambda_i y_{t-i} + \sum_{i=1}^{r} \sum_{j=0}^{s} \beta_{i,j} z_{i,t-j} + \epsilon_t
\]  

(3)

\( x_t = (y_t, z_t) \) could also be modelled as a system:

\[
x_t = \gamma + \sum_{j=1}^{s} \Gamma_j x_{t-j} + \epsilon_t
\]  

(4)

We focus on single equations, but systems can be handled.
Test for non-linearity in general linear model by low-dimensional portmanteau test in Castle and Hendry (2010b) (cubics of principal components $w_t$ of the $z_t$).

(ii) If reject, create $g(w_t)$, otherwise $g(z_t) = z_t$: presently, implemented general cubics with exponential functions.

Number of potential regressors for cubic polynomials is:

$$M_K = K (K + 1) (K + 5) / 6.$$ 

Explosion in number of terms as $K = r \times (s + 1)$ increases:

<table>
<thead>
<tr>
<th>$K$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_K$</td>
<td>3</td>
<td>9</td>
<td>19</td>
<td>30</td>
<td>55</td>
<td>285</td>
<td>679</td>
<td>1539</td>
<td>5455</td>
<td>12300</td>
</tr>
</tbody>
</table>

Quickly reach huge $M_K$: but only $3K$ if use $w_{i,t-j}^k$.

(Investigating squashing functions, to better approximate non-linearity in economics, suggested by Hal White)
(iii) To tackle multiple breaks & data contamination (outliers), add \( T \) impulse indicators to candidates for \( T \) observations.

Consider \( y_i \sim \text{IID} [\mu, \sigma^2] \) for \( i = 1, \ldots, T \)

\( \mu \) is parameter of interest

Uncertain of outliers, so add \( T \) indicators \( 1\{t=t_i\} \) to set of candidate regressors.

First, include half of indicators, record significant:

just ‘dummying out’ \( T/2 \) observations for estimating \( \mu \)

Then omit, include other half, record again.

Combine sub-sample indicators, & select significant.

\( \alpha T \) indicators selected on average at significance level \( \alpha \)

Feasible ‘split-sample’ impulse-indicator saturation (IIS) algorithm: see Hendry, Johansen and Santos (2008)
Johansen and Nielsen (2009) extend IIS to both stationary and unit-root autoregressions.

When distribution is symmetric, adding $T$ impulse-indicators to a regression with $n$ variables, coefficient $\beta$ (not selected) and second moment $\Sigma$:

$$T^{1/2}(\tilde{\beta} - \beta) \xrightarrow{D} N_n \left[ 0, \sigma^2 \Sigma^{-1} \Omega_\beta \right]$$

Efficiency of IIS estimator $\tilde{\beta}$ with respect to OLS $\hat{\beta}$ measured by $\Omega_\beta$ depends on $c_\alpha$ and distribution.

Must lose efficiency under null: but small loss $\alpha T$—only 1% at $\alpha = 1/T$ if $T = 100$, despite $T$ extra candidates.

Potential for major gain under alternatives of breaks and/or data contamination: variant of robust estimation.
Size of the break is 10 standard errors at $0.75T$

There are no outliers in this mis-specified model as all residuals $\in [-2, 2]$ SDs:
outliers $\neq$ structural breaks

step-wise regression has zero power

Let’s see what Autometrics reports
‘Split-sample’ search in IIS

Dummies included in GUM

Final model: actual and fitted

Final model: Dummies selected

Block 1

Block 2

Final
Most major formulation decisions now made: which \( r \) variables (\( w_t \), after transforming \( z_t \)); their lag lengths (\( s \)); functional forms (cubics); structural breaks (any number, anywhere). Leads to general unrestricted model (GUM):

\[
y_t = \sum_{i=1}^{r} \sum_{j=0}^{s} \beta_{i,j} z_{i,t-j} + \sum_{i=1}^{r} \sum_{j=0}^{s} \kappa_{i,j} w_{i,t-j} + \sum_{i=1}^{r} \sum_{j=0}^{s} \theta_{i,j} w_{i,t-j}^{2} + \sum_{i=1}^{r} \sum_{j=0}^{s} \gamma_{i,j} w_{i,t-j}^{3} + \sum_{j=1}^{s} \lambda_j y_{t-j} + \sum_{i=1}^{T} \delta_i 1_{\{i=t\}} + \epsilon_t
\]

\( K = 4r(s + 1) + s \) potential regressors, plus \( T \) indicators: close to what I showed live earlier. Bound to have \( N > T \): consider exogeneity later.
Consider a perfectly orthogonal regression model:

$$y_t = \sum_{i=1}^{N} \beta_i z_{i,t} + \epsilon_t \quad (5)$$

$$E[z_{i,t} z_{j,t}] = \lambda_{i,i} \text{ for } i = j \text{ } \& \text{ } 0 \forall i \neq j, \epsilon_t \sim \text{IN}[0, \sigma^2_\epsilon] \text{ and } T \gg N.$$  

Order the $N$ sample $t^2$-statistics testing $H_0: \beta_j = 0$:

$$t^2_{(N)} \geq t^2_{(N-1)} \geq \cdots \geq t^2_{(1)}$$

Cut-off $m$ between included and excluded variables is:

$$t^2_{(m)} \geq c^2_\alpha > t^2_{(m-1)}$$

Larger values retained: all others eliminated. **Only one decision needed even for $N \geq 1000$:**

‘repeated testing’ does not occur, and ‘goodness of fit’ is never considered.

Maintain average false null retention at **one variable** by $\alpha \leq 1/N$, with $\alpha$ declining as $T \to \infty$.
Repeated testing

Does repeated testing distort selection?

(a) Severe illness: more tests increase probability of correct diagnosis.
Repeated testing

Does repeated testing distort selection?

- (a) Severe illness: more tests increase probability of correct diagnosis.
- (b) Mis-specification tests: if $r$ independent tests $\tau_j$ conducted under null for small significance level $\eta$ (critical value $c_\eta$):

$$P(|\tau_j| < c_\eta \mid j = 1, \ldots, r) = (1 - \eta)^r \approx 1 - r\eta.$$ 

More tests increase probability of false rejection. Suggests significance level $\eta$ of 1% or tighter.
Does repeated testing distort selection?

- (a) Severe illness: more tests increase probability of correct diagnosis.
- (b) Mis-specification tests: if $r$ independent tests $\tau_j$ conducted under null for small significance level $\eta$ (critical value $c_\eta$):

$$P(|\tau_j| < c_\eta \mid j = 1, \ldots, r) = (1 - \eta)^r \simeq 1 - r\eta.$$ 

More tests increase probability of false rejection. Suggests significance level $\eta$ of 1% or tighter.

- (c) Repeated diagnostic tests: probabilities unaltered. Conclude: no generic answer.
Interpretation

Path search gives impression of ‘repeated testing’. Confused with selecting from $2^N$ possible models (here $2^{1000} = 10^{301}$, an impossible task).

We are selecting variables, not models, & only $N$ variables. **But selection matters, as only retain ‘significant’ outcomes.**

Sampling variation also entails retain irrelevant, or miss relevant, by chance near selection margin.

Conditional on selecting, estimates biased away from origin: **but can bias correct as know $c_\alpha$.**

Small efficiency cost under null for examining many candidate regressors, even $N >> T$.

Almost as good as commencing from LDGP at same $c_\alpha$. 
(1) Discovery in general
(2) Automatic model extension
(3) **Automatic model selection**
(4) Automatic model evaluation
(5) Embedding theory models
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**Conclusions**
Autometrics improves on previous algorithms

- **Search paths:** Autometrics examines whole search space; discards irrelevant routes systematically.
- **Likelihood-based:** Autometrics implemented in likelihood framework.
- **Efficiency:** Autometrics improves computational efficiency: avoids repeated estimation & diagnostic testing, remembers terminal models.
- **Structured:** Autometrics separates estimation criterion, search algorithm, evaluation, & termination decision.
- **Generality:** Autometrics can handle $N > T$.

If GUM is congruent, so are all terminals: undominated, mutually-encompassing representations. If several terminal models, all reported: can combine, or one selected (by, e.g., Schwarz, 1978, criterion).
Search follows branches till no insignificant variables; tests for congruence and parsimonious encompassing; backtracks if either fails, till first non-rejection found.
Selecting by Autometrics

Even when 1-cut applicable, little loss, and often a gain, from using path-search algorithm Autometrics. Autometrics applicable to non-orthogonal problems, and $N > T$.

‘Gauge’ (average retention rate of irrelevant variables) close to $\alpha$.

‘Potency’ (average retention rate of relevant variables) near theory value for a 1-off test.

Goodness-of-fit not directly used to select models & no attempt to ‘prove’ that a given set of variables matters, but choice of $c_\alpha$ affects $R^2$ and $n$ through retention by $|t_{(n)}| \geq c_\alpha$.

Conclude: ‘repeated testing’ is not a concern.
Selecting non-linear models

Transpires there are four major sub-problems:
(A) specify general form of non-linearity
(B) non-normality: non-linear functions capture outliers
(C) excess numbers of irrelevant variables
(D) potentially more variables than observations

Have solutions to all four sub-problems:
(A) investigator’s preferred general function, simplified by encompassing tests against specific (ogive) forms
(B) remove outliers by IIS
(C) super-conservative selection strategy
(D) multi-stage ‘combinatorial selection’ for $N > T$

Automatic algorithm for up to cubic polynomials with polynomials times exponentials in Castle and Hendry (2010a).
(1) Discovery in general
(2) Automatic model extension
(3) Automatic model selection
(4) **Automatic model evaluation**
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Conclusions
Under null of congruent GUM, Figure 35 compares ‘gauges’ for Autometrics with diagnostic checking on vs. off:

\[ y_t = \sum_{i=1}^{N} \beta_i z_{i,t} + \epsilon_t \quad \text{for} \quad \epsilon_t \sim \text{IN}[0, \sigma^2] \]  

\( T = 100, \ n = 1, \ldots, 10 = N; \ \beta_k = 0 \text{ for } k > n; \ R^2 = 0.9. \)

‘Gauge’ is average retention rate of irrelevant variables (should be close to \( \alpha \)).

‘Potency’ is average retention rate of relevant variables (should be near theory power for a 1-off test).

Gauge is close to \( \alpha \) if diagnostic tests not checked.

Gauge is larger than \( \alpha \) with diagnostics on, when checking to ensure a congruent reduction.

Difference seems due to retaining insignificant irrelevant variables which proxy chance departures from null of mis-specification tests.
Gauges with diagnostic tests off & on

Gauges for *Autometrics* with and without diagnostics

- **Autometrics (diagnostics) 10%**
- **Autometrics (diagnostics) 5%**
- **Autometrics (diagnostics) 1%**
- **Autometrics (no diagnostics) 10%**
- **Autometrics (no diagnostics) 5%**
- **Autometrics (no diagnostics) 1%**

$$\alpha = 1\%$$

$$\alpha = 5\%$$

$$\alpha = 10\%$$
Role of encompassing

Variables removed only when new model is a valid reduction of GUM.

Reduction fails if result does not parsimoniously encompass GUM at $c_\alpha$: (see Hendry, 1995, §14.6).

If so, variable retained despite being insignificant on t-test, as in Doornik (2008).

*Autometrics* without encompassing loses both gauge and potency:
- gauge is the average retention rate of irrelevant variables;
- potency is average retention rate of relevant variables

*Autometrics* with encompassing is well behaved:
- gauge is close to nominal rejection frequency $\alpha$.
- potency is close to theory maximum of 1-off t-test.
Hoover and Perez (1999) experiments:

**HP7**
\[ y_{7,t} = 0.75y_{7,t-1} + 1.33x_{11,t} - 0.9975x_{11,t-1} + 6.44u_t \]
\[ R^2 = 0.58 \]

**HP8**
\[ y_{8,t} = 0.75y_{8,t-1} - 0.046x_{3,t} + 0.0345x_{3,t-1} + 0.073\lambda u_t \]
\[ R^2 = 0.93 \]

where \( u_t \sim \text{IN}[0, 1] \); \( x_{i,t-j} \) are US macro data

The GUM has 3 DGP variables plus 37 irrelevant. Table [1] shows results for range of values of \( \lambda \) and \( \alpha \) in HP8 (they set \( \lambda = 1 \)).

Later consider 141 irrelevant, larger than \( T = 139 \).
Simulations for encompassing

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\lambda$</th>
<th>Autometrics with encompassing</th>
<th>Autometrics no encompassing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>50</td>
<td>0.093 0.441</td>
<td>0.056 0.402</td>
</tr>
<tr>
<td>0.05</td>
<td>50</td>
<td>0.055 0.405</td>
<td>0.021 0.364</td>
</tr>
<tr>
<td>0.01</td>
<td>50</td>
<td>0.014 0.357</td>
<td>0.002 0.337</td>
</tr>
<tr>
<td>0.1</td>
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<td>10</td>
<td>0.057 0.935</td>
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</tr>
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<td>10</td>
<td>0.017 0.895</td>
<td>0.002 0.630</td>
</tr>
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<tr>
<td>0.01</td>
<td>1</td>
<td>0.014 1.000</td>
<td>0.002 0.999</td>
</tr>
</tbody>
</table>

Table 1: HP8 with $M = 10000$ and $T = 139$. 
Parameter invariance essential in policy models: else mis-predict under regime shifts. **Super exogeneity** combines parameter invariance with valid conditioning so crucial for economic policy.

**New automatic test in** Hendry and Santos (2010): impulse-indicator saturation in marginal models, retain all significant outcomes and test their relevance in conditional model

No *ex ante* knowledge of timing or magnitudes of breaks: need not know DGP of marginal variables

Test has correct size under null of super exogeneity for a range of sizes of marginal-model saturation tests

**Power to detect failures of super exogeneity when location shifts in marginal models**
(1) Discovery in general
(2) Automatic model extension
(3) Automatic model selection
(4) Automatic model evaluation
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Conclusions
Approach is not atheoretic.

Theory formulations should be embedded in GUM, can be retained without selection.

Call such imposition ‘forcing’ variables—ensures they are retained, but does not guarantee they will be significant.

Can also ensure theory-derived \textit{signs} of long-run relation maintained, if not significantly rejected by the evidence.

But much observed data variability in economics is due to features absent from most economic theories: which empirical models must handle.

Extension of LDGP candidates, $x_t$, in GUM allows theory formulation as special case, yet protects against contaminating influences (like outliers) absent from theory.

‘Extras’ can be selected at tight significance levels.
Four possible economic theory outcomes

1] **Theory exactly correct:**
all aspects significant with anticipated signs, no other variables kept.

2] **Theory only part of explanation:**
all aspects significant with anticipated signs, but other variables also kept as substantively relevant.

3] **Theory partially correct:**
only some aspects significant with anticipated signs, and other variables also kept as substantively relevant.

4] **Theory not correct:**
no aspects significant and other variables do all explanation.

Consider these in turn: see Hendry and Johansen (2010).
Theory specifies correct set of \( n \) relevant variables, \( z_t \), with parameters \( \beta \):

\[
y_t = \beta' z_t + \epsilon_t
\]  
(7)

where \( \epsilon_t \sim \text{IN}[0, \sigma_{\epsilon}^2] \), independently of \( z_t \). Then:

\[
\hat{\beta} = \beta + \left( \sum_{t=1}^{T} z_t z_t' \right)^{-1} \sum_{t=1}^{T} z_t \epsilon_t \sim N_n \left[ 0, \sigma_{\epsilon}^2 \left( \sum_{t=1}^{T} z_t z_t' \right)^{-1} \right]
\]  
(8)

Next, \( z_t \) ‘forced’ to be retained during model selection over second set of \( k \) irrelevant candidate variables, \( w_t \), with coefficients \( \gamma = 0 \) when \((k + n) << T\), so GUM is:

\[
y_t = \beta' z_t + \gamma' w_t + \nu_t
\]  
(9)

Orthogonalize \( z_t \) and \( w_t \) by:

\[
w_t = \hat{\Gamma} z_t + u_t
\]  
(10)

Then as \( \gamma = 0 \):

\[
y_t = \beta' z_t + \gamma' w_t + \nu_t = \beta' z_t + \gamma' u_t + \nu_t
\]  
(11)
Consequently:

\[
\begin{pmatrix}
\tilde{\beta} - \beta \\
\tilde{\gamma}
\end{pmatrix} = \begin{pmatrix}
\sum_{t=1}^{T} z_t z'_t & \sum_{t=1}^{T} z_t u'_t \\
\sum_{t=1}^{T} u_t z'_t & \sum_{t=1}^{T} u_t u'_t
\end{pmatrix}^{-1} \begin{pmatrix}
\sum_{t=1}^{T} z_t \nu_t \\
\sum_{t=1}^{T} u_t \nu_t
\end{pmatrix}
\]

\[
\sim N_{n+k} \left[ \begin{pmatrix}
0 \\
0
\end{pmatrix}, \sigma^2_{\epsilon} \begin{pmatrix}
\left( \sum_{t=1}^{T} z_t z'_t \right)^{-1} & 0 \\
0 & \left( \sum_{t=1}^{T} u_t u'_t \right)^{-1}
\end{pmatrix} \right]
\]

(12)

as \( \sum_{t=1}^{T} z_t u'_t \sim 0 \), so distribution of \( \tilde{\beta} \) in (12) identical to that of \( \hat{\beta} \) in (8): unaffected by model selection.

Only costs of selection are:
(a) chance retentions of some \( u_t \) from selection; and
(b) impact on estimated distribution of \( \tilde{\beta} \) through \( \tilde{\sigma}^2_{\epsilon} \). Can be offset by bias correction.
Different when **theory model is only part of explanation**: defined as all aspects significant with anticipated signs, but other variables also kept as substantively relevant.

Two distinct forms of under-specification:

a] omitting relevant functions or lags of variables in LDGP; avoided by sufficiently general initial model:

b] omitting relevant variables, \( w_t \), from the DGP; induces less useful LDGP—hard to avoid if \( w_t \) unknown.

In a], \( \gamma \neq 0 \), as \( z_t \) and \( u_t \) orthogonal in (13), coefficient of former is \( \beta + \gamma' \hat{\Gamma} \), which is estimated if (7) is simply fitted to the data: but may be significant with anticipated signs.

In b], when (9) nests LDGP, but \( w_t \) omitted from DGP, selection can substantively improve the final model: (see Castle and Hendry, 2010c), as we will show.
Next, when the theory is only partially correct: some aspects significant with anticipated signs, but other aspects not significant, or ‘wrong’ signed, with other variables also kept as substantively relevant. Under alternative, $\gamma \neq 0$, estimating (7) will result in biased, inefficient, possibly non-constant, estimates as:

$$y_t = \beta' z_t + \gamma' (\hat{\Gamma} z_t + u_t) + \nu_t = (\beta + \gamma' \hat{\Gamma})' z_t + \gamma' u_t + \nu_t$$

(13)

Now forcing $z_t$ when selecting from (13) will deliver an incorrect estimate of $\beta$, but some of the $u_t$ will be correctly retained, so an implied estimate of $\beta$ can be derived from $\beta + \gamma' \hat{\Gamma}$, $\tilde{\gamma}$ and $\hat{\Gamma}$. A better estimate of $\tilde{\sigma}_\nu^2$ should result.

Selection can also help when relevant variables, $w_t$, omitted from DGP and breaks occur.
Finally, theory is now completely incorrect: no aspects significant and other variables do all explanation.

Despite forcing $z_t, \beta = 0$, but interpretation awkward as coefficient of $z_t$ is $\gamma \hat{\Gamma}$.

Win-win situation: theory kept if valid and complete; yet learn when it is not correct—empirical model discovery embedding theory evaluation.

Interesting case is when $N > T$ for $N$ candidates, so can automatic model selection work then?
(1) Discovery in general
(2) Automatic model extension
(3) Automatic model selection
(4) Automatic model evaluation
(5) Embedding theory models
(6) **Excess numbers of variables** \( N > T \)
(7) Non-invariance of NKPCs

Conclusions
As many candidate variables as observations

Analytic approach to understanding IIS applies for $N = T$ IID mutually orthogonal candidate regressors under the null.

Add first $N/2$ and select at significance level $\alpha = 1/T = 1/N$. Record which were significant, and drop all.

Now add second block of $N/2$, again select at significance level $\alpha = 1/N$, and record which are significant.

Finally, combine recorded variables from the two stages (if any), and select again at significance level $\alpha = 1/N$.

At both sub-steps, on average $\alpha N/2 = 1/2$ a variable will be retained by chance, so on average $\alpha N = 1$ from the combined stage.

Again 99% efficient under the null at eliminating irrelevant variables—lose one degree of freedom on average.
More candidate variables than observations

If also have relevant variables to be retained, and $N > T$, orthogonalize them with respect to the rest.

As $N > T$, divide in more sub-blocks, setting $\alpha = 1/N$.

Basic model retains desired sub-set of $n$ variables at every stage, and only selects over putative irrelevant variables at stringent significance level:

under the null, has no impact on estimated coefficients of relevant variables, or their distributions.

Thus, almost costless to check even large numbers of candidate variables:

huge benefits if initial specification incorrect but enlarged GUM nests LDGP.
## Table 2: IIS for breaks in *Autometrics*

<table>
<thead>
<tr>
<th>1% nominal size</th>
<th>( \gamma = 0 )</th>
<th>( \gamma = 1 )</th>
<th>( \gamma = 2 )</th>
<th>( \gamma = 3 )</th>
<th>( \gamma = 4 )</th>
<th>( \gamma = 5 )</th>
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</thead>
<tbody>
<tr>
<td><strong>D1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gauge %</td>
<td>1.5</td>
<td>1.2</td>
<td>0.9</td>
<td>0.3</td>
<td>0.7</td>
<td>1.1</td>
</tr>
<tr>
<td>Potency %</td>
<td>—</td>
<td>4.6</td>
<td>25.6</td>
<td>52.6</td>
<td>86.3</td>
<td>99.0</td>
</tr>
<tr>
<td>DGP found %</td>
<td>29.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>8.1</td>
<td>36.8</td>
</tr>
<tr>
<td><strong>D2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gauge %</td>
<td>1.5</td>
<td>1.0</td>
<td>0.4</td>
<td>0.3</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Potency %</td>
<td>—</td>
<td>3.5</td>
<td>7.9</td>
<td>24.2</td>
<td>67.1</td>
<td>90.2</td>
</tr>
<tr>
<td>DGP found %</td>
<td>29.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>3.9</td>
<td>24.2</td>
</tr>
</tbody>
</table>
**Hoover–Perez experiments**

\( T = 139, \) **3 relevant and 37 irrelevant variables**

<table>
<thead>
<tr>
<th></th>
<th>Hoover–Perez HP7</th>
<th>Hoover–Perez HP8</th>
<th>step-wise HP7</th>
<th>step-wise HP8</th>
<th>Autometrics HP7</th>
<th>Autometrics HP8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1% nominal size</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gauge %</td>
<td>3.0*</td>
<td>0.9*</td>
<td>0.9</td>
<td>3.1</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Potency %</td>
<td>94.0</td>
<td>99.9</td>
<td>100.0</td>
<td>53.3</td>
<td>99.2</td>
<td>100.0</td>
</tr>
<tr>
<td>DGP found %</td>
<td>24.6</td>
<td>78.0</td>
<td>71.6</td>
<td>22.0</td>
<td>68.3</td>
<td>68.8</td>
</tr>
</tbody>
</table>

* Only counting significant terms (but tiebreaker is best-fitting model)

\( T = 139, \) **3 relevant and 141 irrelevant variables**

<table>
<thead>
<tr>
<th></th>
<th>step-wise HP7</th>
<th>step-wise HP8</th>
<th>Autometrics HP7</th>
<th>Autometrics HP8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>0.1% nominal size</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gauge %</td>
<td>0.1</td>
<td>0.7</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Potency %</td>
<td>99.7</td>
<td><strong>40.3</strong></td>
<td>97.4</td>
<td><strong>100.0</strong></td>
</tr>
<tr>
<td>DGP found %</td>
<td>87.4</td>
<td><strong>9.0</strong></td>
<td>82.9</td>
<td><strong>90.2</strong></td>
</tr>
</tbody>
</table>

**Large increase in probability of locating DGP relative to** \( \alpha = 0.01 \)

**not monotonic in** \( \alpha \) – so should not select by ‘goodness of fit’
(1) Discovery in general
(2) Automatic model extension
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(7) Non-invariance of NKPCs

Conclusions
Non-invariance of NKPCs

‘Hybrid’ NKPC given by:

\[
\Delta p_t = \gamma f E_t [\Delta p_{t+1} | I_t] + \gamma b \Delta p_{t-1} + \pi s_t + u_t \geq 0
\]

\[
\Delta p_t, s_t \text{ are rate of inflation & firms’ real marginal costs, so:}
\]

\[
\Delta p_t = \gamma f \Delta p_{t+1} + \gamma b \Delta p_{t-1} + \pi s_t + \epsilon_t, \quad \epsilon_t \sim D [0, \sigma^2_{\epsilon}]
\]

where it is claimed:

\[
E_t [\Delta p_{t+1} | I_t] = \Delta p_{t+1} + \nu_{t+1}
\]

\[
\Delta p_{t+1} \text{ instrumented by } k \text{ variables } \mathbf{z}_t \text{ implicitly postulating:}
\]

\[
\Delta p_t = \kappa' \mathbf{z}_t + v_t
\]

Assumes a constant world. Test by IIS on (17) adding indicators to (15): significance refutes invariance. Also, insignificance of \( \tilde{\gamma}_f \) inconsistent with forward-looking formulation.
Euro-area hybrid NKPC with IV estimation, $\Delta p_{t+1}$ and $s_t$ endogenous, using instruments: five lags of inflation, two lags of $s_t$, detrended output and wage inflation; sample $T = 102$ (1972(2) to 1998(1)):

$$\hat{\Delta} p_t = 0.655 \hat{\Delta} p_{t+1} + 0.280 \hat{\Delta} p_{t-1} + 0.012 s_t + 0.009$$

$$\chi^2_S(6) = 11.88$$

Elasticities sum to 0.94 and $\hat{\gamma}_f$ comparable to reported GMM estimates.
Forecasting equation for $\Delta p_t$ uses instrument set for NKPC estimation with IIS in *Autometrics*

For $\alpha = 0.025$, finds 11 indicators.

When hybrid NKPC augmented by these, non-congruent, with $\chi^2_S(6) = 17.83^{**}$.

Some of instruments have explanatory power for $\Delta p_t$, consistent with (earlier) standard models of inflation.

Adding $gap_{t-1}$ and the 11 indicators makes NKPC congruent: $\chi^2_S(4) = 2.42$.

No significant tests of residual mis-specification.
Selecting by *Autometrics* with $\alpha = 0.05$:

$$\hat{\Delta}p_t = -0.325 \hat{\Delta}p_{t+1} + 0.117 s_t + 0.515 \Delta p_{t-1} + 0.088 + 0.0016 \text{ gap}_{t-1}$$

$$+ 1.10 I_{73(1),t} + 1.12 I_{73(3),t} + 0.74 I_{73(4),t} + 0.87 I_{74(2),t}$$

$$+ 0.82 I_{74(3),t} + 1.01 I_{76(2),t} + 0.56 I_{76(3),t} - 0.67 I_{78(4),t} + 0.69 I_{83(1),t}$$

$$\chi^2_S(6) = 5.06 \quad F_{ar}(5,85) = 1.55 \quad F_{arch}(4,82) = 1.27$$

$$F_{het}(17,72) = 0.91 \quad \chi^2_{nd}(2) = 1.04$$

(coefficients of dummies multiplied by 100)

$F_{name}$ denotes an approximate F-test:

- $F_{ar}$ for $k^{th}$-order serial correlation;
- $F_{het}$ for heteroskedasticity;
- $F_{reset}$ for functional form;
- $F_{arch}$ for $k^{th}$-order ARCH; and
- $\chi^2_{nd}(2)$ for normality.
Nine of the 11 ‘reduced form’ indicators retained: clear evidence for lack of invariance in feed-forward NKPC. 

Coefficient of $\Delta p_{t+1}$ is negative and insignificantly different from zero.

Coefficient of wage-share is sizeable, serves as an important equilibrating mechanism.

A failure to model breaks induces spurious significance of feed-forward terms proxying expectations

Inflation ‘persistence’ seems an artifact of mis-specified NKPC models
(1) Discovery in general
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Conclusions
Conclusions

All essential steps feasible once target LDGP defined:

1. automatically create general model from investigator’s $x_t$: extra variables, lags, non-linearity, & impulse indicators—ensures congruent GUM;
2. embed theory-model as a ‘forced’ specification—ensures theory insights retained;
3. select most parsimonious encompassing model—ensures undominated representation;
4. compute near-unbiased parameter estimates—ensures appropriate policy analyses; and
5. stringently evaluate results—ensures selected model valid.

Generalizes to $N > T$ with expanding and contracting searches: see HP8 when $N = 145, T = 139$ at $\alpha = 0.001$. 

David F. Hendry

Empirical Model Discovery and Theory Evaluation – p.60/64
Overall conclusions

Little difficulty in eliminating almost all irrelevant variables from the GUM (a small cost of search). Avoids huge costs from under-specified models.

When the LDGP would be retained by *Autometrics* if commenced from it, then a close approximation is generally selected when starting from a GUM which nests that LDGP.

Model selection by *Autometrics* with tight significance levels and bias correction is a successful approach which allows multiple breaks to be tackled.

Applied to NKPC shows lack of invariance, insignificance of feed-forward term

All the ingredients for empirical model discovery jointly with theory evaluation are in place.
Host of developments in automatic empirical model discovery already achieved

Theory of many stages still to be formalized

Now implementing *automatic*:
- modelling of simultaneous systems
- selecting cointegration vectors
- testing expectations models for invariance
- model averaging across terminals for forecasting.

The future is bright:

the future is *Autometrics*
References


Retracing route

(1) Discovery in general
(2) Automatic model extension
(3) Automatic model selection
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Conclusion