Centre for Global Finance
Working Paper Series (ISSN 2041-1596)
Paper Number: 02/11

Title:

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Using a Simplified Miles-Ezzell Framework to Value Equity

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Abstract

The paper presents a unique demonstration of the adjusted present value (APV) method of equity valuation. Using theoretical and practical evidence, it argues that the unlevered cost of equity should be used to discount the tax shield in all forecast periods. Several other methods of equity valuation are also presented and their equivalence to APV is shown. It is argued that this simplified framework, based on the work of Miles and Ezzell (1985), works with any leverage forecast and should be of interest to investment analysts, financial managers and business valuers.

Keywords – Discounted cash flow, valuation, adjusted present value

Using a Simplified Miles-Ezzell Framework to Value Equity

I. Introduction and Background

This paper presents a framework for the valuation of equity using a method first proposed by Miles and Ezzell (1980, 1985), who used an unlevered cost of equity to value tax shields within an adjusted present value valuation approach. The adjusted present value (APV) equity valuation method is claimed to be superior to other discounted cash flow (DCF) methods of valuation. For example, Sabal (2007) states "APV has certain advantages making it more convenient for smaller companies with unstable debt ratios, in countries with complex tax legislation and in emerging markets where high economic uncertainty makes the leveraging decision much more opportunistic" (p.2) and Esty (1999) argues that APV is the preferred method for firms that are likely to substantially change their capital structure and is useful in leveraged management buyouts and large investments that require changes in capital structure. It was first proposed by Myers (1974) and has generated a great deal of academic interest since this date (see, for example, Ashton and Atkins (1978), Inselbag and Kaufold (1989), Arzac (1996) and Qi (2010)). Theoretically, is recognised as "intellectually" superior to other methods based on WACC, for example Sweeney (2002) recommends the APV method as a starting point because "the WACC approach is a special case of the APV approach" (p.15). Luehrman (1997) goes further and claims that "adjusted present value is especially versatile and reliable, and will replace WACC as the DCF methodology of choice among generalists" (p.145). Despite its popularity in the theoretical finance literature, the use of APV in industry seems to be less well developed. For example, APV is not identified as being used in surveys of investment analyst's valuation methods (for example, Arnold and Moizer (1984), Pike et al. (1993), Barker (1999), Demirikos et al (2004, 2010), Imam at al (2008)). However, some empirical support for APV is provided by Kaplan and Ruback (1995), who link cash flow forecasts with market values via a 'capital cash flow' or 'compressed' APV valuation model. APV is the only method that identifies the value of the tax shield separately and can be used to see if debt has any value in corporate valuation. The importance of debt in a firm's capital structure is well known although its affect is uncertain. There is an empirical literature linking the value of a firm's debt to its corporate value (for example, Bowman 1979, Fama and French 1998, Engel et al (1999)) which on balance appears to conclude that the use of debt e.g. a debt for equity swap, tends to increase equity value.

Booth (2002) identifies several disadvantages of APV including its failure to effectively value distress costs, agency costs and personal taxation. Distress costs and personal taxation need to be included to counteract the corporate income tax advantage of debt as the APV valuation method will otherwise put a too high value on firms with tax shields. Such issues have been fully considered in theory by Cooper and Nyborg (2008) but applying APV fully in practice is problematic as personal tax rates and distress cost triggers need to be specified. Authors such as Ehrhardt and Daves (1999) when applying APV recognize these complications but then assume that the tax shield is the only additional cash flow to consider. The author cannot find a simple solution to the inclusion of personal tax and distress costs and proposes an alternative solution – that the APV method should *only* be considered as one of a number of equity valuation methods. In this paper, the cost of simplifying the APV method is counteracted by showing that the APV result is equivalent to results from other valuation methods. The robustness of any APV result can then be considered by relating to the result with those from other equity valuation methods. For example, the WACC discount rates can be assessed for reasonableness.

APV is one of several methods that can be used to value equity. This paper shows that the APV result is equivalent to valuations from four other methods - the dividend discount method (DDM), the discounted residual income method (DRI), the discounted free cash flow method (DFCF) and the discounted firm residual income method (DFRI) (sometimes called EVA®, see Stewart 1991). DDM and DRI value equity directly using equity cash flows and levered equity discount rates. DFCF, DFRI and APV value equity indirectly by first finding a firm value and then deducting the value of debt to yield an equity value. DFCF and DFRI use the weighted average cost of capital (WACC). Each of these different equity valuation methods appears to have its proponents. Some authors prefer accounting-based valuation (Penman 2004), others prefer 'free cash flow' methods (Soffer and Soffer, 2003) and others prefer methods based on dividends (Barker 2001). This paper applies the argument presented in Lundholm and O'Keefe (2001a) (henceforth LOK) who argue that *all methods are equivalent*. This is sometimes called 'the law of one price', that is, that value cannot be increased by altering the way in which it is measured. The debate between Penman and Lundholm and O'Keefe in the Winter of 2001 highlights some of the issues surrounding whether all equity valuation methods are theoretically and empirically equivalent (Lundholm and O'Keefe 2001b, Penman 2001). Empirical tests tend to support one equity valuation method over another (see for example, Penman and Sougiannis (1998), Dechow et al (1999)) so the debate is far from resolved.

There is a theoretical mathematical literature demonstrating the equivalence of the various methods of equity valuation, for example Taggart (1977), Lewellen et al (1977), Shrieves and Wachowicz (2001), Lundholm and O'Keefe (LOK) (2001), Sweeney (2002), Fernandez (2002), Oded (2007) and Massari et al (2007). This paper shows the equivalence of several different equity valuation methods when one starts with an APV valuation. LOK present a full discussion and a mathematical reconciliation of DDM, DRI, DFCF and DFRI and their paper is used as the basis for the formulae stated in appendix one. This paper contributes to LOK in that it adds APV to the suite of valuation methods. The interaction between APV and the other methods is important in assessing the validity of some of the decisions that need to be made in using APV. For example, APV will tend to overstate the value of debt tax shields by assuming that changes in equity risk is linearly related to leverage. The method used to relate APV to the other methods is the theme of this paper and the author believes that it is the first time that an APV method has been used in conjunction with the other equity valuation methods. As such, it should be of interest to analysts and business valuers.

The paper proceeds as follows: Section II presents the main theoretical issues that need to be considered in adopting APV. Section III presents the main practical issues that need to be addressed when forecasting and discounting APV valuations. Section IV presents some examples that demonstrate the framework and section V concludes.

II. A Simplified Miles-Ezzell Framework - Theoretical Issues

There are many different APV methods in the literature and in valuing the tax shield the business valuer has to make three major theoretical decisions – the effective rate of tax to apply, the size of the tax shield in a period and the discount rate to derive the present value of the benefit of debt. These three problems will now be discussed in turn.

(i) The Effective Tax Rate

The first decision relates to the effective tax rate. There are three taxes to consider – the firm income tax rate, the personal income tax rate and the personal capital gains tax rate. Miller (1977) was the first author to include personal taxes in the discussion of firm capital structure, with the argument that the benefit of the firm tax shield might be offset with additional personal taxation making capital structure irrelevant when all taxes are considered. Most writers agree that the effective rate of tax is less than the marginal firm income tax rate. Some writers cannot specify the effective rate of tax (see Brealey and Myers 2006 p.539) whilst other writers do specify the effective tax rate that combines firm and personal taxation (Soffer and Soffer (2003) p. 160). For a full discussion of personal taxes, see Cooper and Nyborg (2004 and 2008). This paper does include personal taxes when comparing the various formulae for unlevering equity betas (formula 5, below) but the specification of personal taxes is problematic in practice as the business valuer would need to know the personal income tax situation of the firm's investors. Therefore this paper assumes that only firm income taxes are considered. However, as the firm income tax is an input into the equity valuation framework, it could be reduced to take account of personal taxation but any such reduction would be a matter of professional judgement.

The next decision relates to timing differences. This paper uses the term effective firm income tax (EFIT). The EFIT rate depends upon the profitability of companies - companies do not receive negative payments of firm income tax if losses are made or firm income tax is reduced due to tax breaks. If taxes are reduced the benefit of debt is also reduced. The EFIT rate will therefore vary over time as levels of taxable profits change. Therefore this paper uses the following formula for the calculation of the effective tax rate:

EFIT = Firm income tax payable/profit before tax (1)

If the company is making profits and there are no losses, tax breaks or deferred tax timing issues then the EFIT will equal the standard firm income tax rate. However, in countries where deferred tax is important, such as the UK, the EFIT rate will only equal the standard firm income tax rate when timing differences balance out. The reversal of the timing difference will depend on the company's investment programme, the depreciation rate, the capital allowance rate, tax losses brought forward and the firm income tax rate. In addition, if a firm is making losses that cannot be carried forward the value of the tax shield on debt is zero. Example 3 in appendix 2 shows how tax losses carried forward affects value – the EFIT rate is reduced to zero when the firm is making losses, therefore there is no debt tax shield in the loss making period.

(ii) Measuring the Tax Shield for a Particular Period

The second decision relates to how the value of the tax shield is calculated for a particular period. Most textbooks follow Myers (1974) in applying the cost of debt to the debt value and multiplying by a tax rate to give the tax shield for a particular period. For example, if a company has $400,000 of debt, EFIT rates are 30% and interest is paid at 5% then the value of debt tax shield for a particular period is $6,000 per annum ($400,000*30%*5%). However, Damodaran (1994) applies an unlevered cost of equity less a debt premium after tax to the debt value to measure the tax shield. Booth (2002) and Fernandez (2004) apply the unlevered cost of equity to the debt value, however, Wonder (2005) and Cooper and Nyborg (2006) severely criticise Booth's and Fernadez's approach and state that the use of an unlevered equity cost to calculate the tax shield for a particular period is a surprising choice that lacks theoretical justification and conveniently counteracts a mistake in the authors' choice of discount rate. Fernandez (2005) counters these criticisms. This paper follows the conventional wisdom of Myers and uses the debt cost and EFIT rate to measure the tax shield for a period. Therefore we apply the cost of debt to the opening debt value multiplied by the EFIT rate to get the value of the tax shield for a particular period.

(iii) The Choice of Discount Rate

There are five possible discount rates that can be chosen when discounting the tax shield. All are compatible with the APV method. They are the cost of debt, the unlevered cost of equity, the levered cost of equity, mixtures of these three over time and the weighted average cost of capital (WACC). All methods assume that the capital asset pricing model (CAPM) (Sharp 1963) holds, including the cost of debt which includes a debt beta. Obviously, there is a huge literature on CAPM which is outside the scope of this paper. Whether changes in risk are non-linear, the determination of risk premia and a risk-free rate that are used as inputs into CAPM are likewise unable to be discussed.

Firstly, the cost of debt can be used as the discount rate. In using the cost of debt, Modigliani and Miller (MM) (1963) assume that the risk of the tax shield is closely tied to the value of forecasted debt. This reflects the view that the tax shield is no more or less risky than the interest payments upon which the tax shield depends. MM assume that operating cash flow is a risky perpetuity and that debt leverage is constant. The level of debt is thus fixed, so tax savings from interest have the same risk as the debt upon which interest is paid. Using a MM approach when leverage changes is therefore problematic.

Secondly, an unlevered cost of equity can be used. It is this approach which is used in this paper. In using the unlevered cost of equity Harris and Pringle (1985) assume that the risk of the tax shield is closely tied to the risk of forecasted operating assets. We are therefore assuming that any tax savings from debt have the same risk as the operating cash flows. The approach is adopted by Ruback (1995 and 2000), who calls the approach ‘capital cash flow’ valuation (CCFV) and in Kaplan and Ruback (1995) who call the approach ‘compressed APV’ (CAPV). CCFV and CAPV are the same and use the unlevered cost of equity to value the combined amounts of free cash flows and tax benefits. However, the CCFV and CAPV approaches suffer from the inability to separately disclose the value of the tax shield, a problem that is resolved in the framework proposed in this paper where the valuation of the tax shield and of the free cash flows are separately identified. Assuming that the risk of tax shields is the same as operational cash flows allows forecasts to have any leverage situations thus avoiding the restrictive assumption of constant leverage required for a MM solution. This approach requires that firm leverage is forecast in advance, rather than being a constant ratio but this requirement does not seem to be too onerous given that the requirement to forecast financial statements are part of the equity valuation process. The use of an unlevered cost of equity will produce a lower tax shield value than under MM as the cost of debt is normally less than the cost of equity. Therefore the use of unlevered costs may understate the value of the tax shield for a very stable profitable, low growth firm (although such factors as industry stability could be reflected in the unlevered beta value).

There appears to be a growing acceptance that the unlevered cost of equity is the most suitable discount rate when discounting the tax shield. For example, the approach is beginning to find its way into equity valuation textbooks - Koller et al (2010) present the two approaches and argue that using the unlevered cost of equity is preferable in *all* circumstances, including situations where debt is fixed (see page 124). There is also academic literature suggesting that using the unlevered cost of equity is preferable. Harris and Pringle (1985) developed the work of Miles and Ezzell (1980) and were the first to argue that the unlevered cost of equity should be used for all periods. Tham and Velez-Pareja (2001) argue that the unlevered cost of equity should be used in an N-period case. Ehrhardt and Daves (2002) compare using the debt rate used in Modigliani and Miller (1963) and Hamada (1972) with the unlevered rate used in Kaplan and Ruback's (1995) compressed APV (CAPV) model and conclude that;

"We show that if tax shields of a growing firm are discounted at a rate less than the unlevered cost of equity, then two unusual results are likely to occur. First, the levered cost of equity can actually be smaller than the unlevered cost of equity! Second, the cost of capital decreases as growth increases, implying that high growth firms should have large amounts of debt in their capital structures. These two results are inconsistent with intuition and observation of actual practices, and suggest that the CAPV is the appropriate model with tax shields being discounted at the unlevered cost of equity" (p.1).

Gamba et al. (2008) examine the interaction between personal tax rates and corporate tax rates and state that "the appropriate tax-adjusted discount rate (TADR) is the riskless equity rate of return, $r\ z$, no matter what its capital structure is. In particular, it is inappropriate to discount the interest tax shield at a debt rate, which is an error commonly seen in the literature" (p.43). More recently, Velez-Pareja (2010) has shown via Monte Carlo simulations that using an unlevered cost works most effectively.

Thirdly, the levered equity rate can be used. Kolari (2010) and Tham and Velez-Pareja (2010) use a levered equity discount rate to value the tax shield but offer no theoretical justification for doing so. Presumably, the assumption is that the risk of the tax shield is equal to the risk of the cash flows to equity. This seems unlikely, given that cash flows to equity are a 'bottom line' or the residual after investment in non-current assets and working capital etc.

Fourthly, a mixed approach can be used. Miles and Ezzell (ME) (1985) present a mixed approach where the cost of debt is used to discount the tax shield in the first forecast period and the unlevered cost of equity is used for all subsequent periods, thus assuming that the cash flows have the same risk as debt in the first forecast period and the risk of unlevered equity in subsequent periods. In a similar vein, Luehrman (1997) argued that the correct discount rate should lie somewhere between the cost of debt and the cost of unlevered equity. Given the probable insignificant effect on value, a 'simplified' Miles-Ezzell framework is adopted in this paper where the unlevered cost of equity is applied in all forecast periods.

Finally, it should be noted that the DFCF method uses WACC to discount free cash flows. Although the measurement of tax shields is not separately identified, WACC includes the after tax cost of debt and therefore DFCF implicitly discounts the tax shield at WACC. However, using WACC in this way assumes that there is constant proportional market-value leverage. WACC can be improved by using target leverage or iterative recalculation as leverage changes. In this paper, WACC is not an input, it is an output once an APV value is derived and is rebalanced over time as values change.

Looking at the a-priori arguments and simulations discussed above, it seems as though the unlevered cost of equity is the best choice if forecast operational cash flows and leverage levels are not constant. Empirical evidence is unclear. Kemsley and Nissim (2002) find evidence that

β_U

supports MM and estimate that the value of tax shields are “generally equal to approximately 40 per cent of debt balances”, when corporate tax rates were 45% (page 2047). However, Arzac and Glosten (2005) review the recent research on tax shield valuation and argue that values are normally overstated in practice if the cost of debt is used because of the restrictive MM assumptions about leverage.

Once the decision to use the unlevered cost of equity has been taken, a formula that links the levered and unlevered costs of equity needs to be chosen. Unless a firm has no debt, the unlevered cost of equity is unobservable and given that APV is only useful when there are tax shields to value and to have a tax shield a firm must have debt, the identification of the unlevered cost of equity is problematic. In other words business valuers can only observe levered betas which include business risk and financial structure risk but for APV need a discount rate for the same firm with no leverage. Other firms in the same industry without debt could be used as proxies but given the literature on capital structure using such firms might be problematic – for example, if industry average leverage is 40% a firm with no debt may be trading at a discount because it has not adopted an efficient capital structure. It is therefore normally recommended that average levered equity betas in an industry are adjusted by average industry leverage to get an industry asset or unlevered equity beta. Alternatively the firm’s own observed levered cost of equity could be used. Either way, a formula needs to be chosen that levers and unlevers costs of equity. There are five alternatives – a weighted average of debt after tax and equity betas, a weighted average of debt and equity, a weighted average that assumes the beta of debt is zero, a formula that includes personal taxes and a formula that requires the calculation of the value of the unlevered firm.

Firstly, MM use the following familiar formula for unlevering equity betas:

$$\beta_L = \beta_U + (\beta_U - \beta_d)*(D(1-T)/E) \qquad (2)$$

Where:

β_L = Beta of the equity of the levered company

β_U = Beta of the equity of the unlevered company

β_d = Beta of the debt of the levered company

E = Market value of the equity of the levered company

D = Market value of the debt of the levered company

T = Firm Income Tax rate

Secondly, ME (1985) use the following formula for second and subsequent periods and it is this approach that is adopted in this paper for all periods:

$$\beta_L = \beta_U + (\beta_U - \beta_d)*(D/E) \quad (3)$$

Thirdly, Goetzmann (no date) and Arnold (2005 p.996) follow Hamada (1972), by simplifying equation (3) by assuming that debt has a zero beta:

$$\beta_L = \beta_U + (\beta_U)*(D/E) \quad (4)$$

Therefore the levered equity beta has no link to the debt beta which seems rather simplistic.

Fourthly, following Miller (1977), Cooper and Nyborg (2008, formula 29 page 375) include personal taxes in their unlevered formula, which can be restated thus:

$$\beta_L = \frac{\beta_U + \beta_D.(D/(D+E)).(1\text{-}Tc/1\text{-}T^*)}{E/(E+D)} \quad (5)$$

Where:

Tc is the firm income tax rate

T* is a combination of personal income taxes and capital gains taxes

Although this is the most sophisticated solution of the five presented, the identification of the personal taxes of the firm’s investors is problematic in practice. Appendix 2, example 1, presents a comparison of these different formulae and assumes a personal income tax rate of 22%.

Fifthly, Tham and Velez-Pareja (2010) uses Ke to discount that tax shield which uses the following formula:

$$\beta_L = \quad \beta_U + (\beta_U - \beta_d)*(D/Vu - D) \qquad (6)$$

Where Vu is the value of the unlevered firm. This creates a circular argument as to value of the unlevered firm an unlevered equity beta is required so Tham and Velez-Pareja use the DDM to get an equity value which is then plugged back into the formula. As shown in appendix 2, example 1, the equity value from Tham and Velez-Pareja's reveals no additional information as the value of the unlevered cash flows is a balancing figure once the equity value is taken from the DDM and the tax shield is discounted at the levered cost of equity. It is thus included in this paper for completeness and does not represent a viable alternative to other options as the approach lacks any theoretical justification.

These five formulae for levering/unlevering equity betas are presented in example 1 in Appendix 2, with the additional assumptions of a levered equity beta of 1.4 and a personal tax rate of 22%. It can be seen that from the five alternative formulae, four different equity values emerge. With formula (6), Tham and Velez-Pareja (2010) generate the value of free cash flow as a balancing figure, therefore one result appears twice. Formula (3) gives the same equity value as shown in the detailed calculations as it is this formula that is used in the rest of the paper. Formula (2) gives the highest value of the tax shield because it is discounted at the lowest rate, R_d. Of the five formulae presented, the Miles-Ezzell formula appears to be a compromise between the simplicity of assuming debt has a zero beta and the complexity of including personal taxes.

The main theoretical decisions that are required in the application of a simplified ME APV valuation have now been discussed. There are now a number of practical issues that need to be resolved.

III. A Simplified Miles-Ezzell Framework – Practical Issues

(i) The definition of Free Cash Flow, other Cash Flows and Discount Rates

The formulae used are shown in Appendix 1 and have been adapted from LOK. APV discounts the cash flow of a hypothetical unlevered company at the unlevered cost of equity to yield the value of the firm as if there were no debt. To this the value of the tax shield is added and the value of debt is subtracted to give the value of equity. The calculation of the tax shield cash flow in the terminal period requires thought as it is not the same as the cash flow used in the FCF valuation because the accounting variables grow at different rates.

(ii) The Identification of Unlevered or Asset Betas

Levered equity betas are observable in the market place and can be easily obtained from well-known data sources such Datastream, Thomson Reuters or London Business School's 'Risk Measurement Service'. Using the APV method relies on knowledge of an unobservable unlevered equity beta – no one knows what the beta of a specific company would be if the debt of this company were removed. However, unlevered betas from other similar businesses could be used and by comparing industry leverage and levered betas, unlevered betas can be identified using formula (3). It is this approach that is used in the framework proposed, namely an industry business risk as represented by an unlevered beta is an input into the valuation model.

(iii) Time Varying and Intrinsic Value Dependant Levered Betas

To avoid the circularity problem (Velez-Pareja 2008) it is assumed that unlevered equity betas are taken from industry data. APV therefore gives an intrinsic equity value which is used as an input into the other valuation methods. These intrinsic values are important as they are dynamic and are used as inputs into the other valuation methods, for example, WACC is based on levered equity returns, weighted by the intrinsic value of equity and debt from the APV valuation. Equity (E) in formula (3) is therefore an intrinsic value, rather than a value from the market. Market values of equity cannot be used if equivalent results are to be presented, for example WACC is based on an intrinsic APV value rather than market values. The framework uses APV to generate the value of equity at t_0 which is then rolled forward over the forecast time periods. Likewise, debt's intrinsic value might differ to its book value and there may exist a residual income on debt. However, for simplicity, debt has been kept at book value in this paper.

One contribution of this paper to the debate on equity valuation methods is to show that the iterative search for a solution where each equity valuation method gives the same answer is not necessary. In this paper the value of equity at t_0 is taken from the APV method and it is this value that is used as an input for the other methods. Unfortunately some finance texts rely on this iterative search, for example, Soffer and Soffer (2002) state that students should assume that all surplus cash is invested in projects with a zero NPV - when NPV depends on WACC which in turn depends on NPV values. Another example is Copeland et al (2000) where the APV value is 1% different to the DFCF value and an iterative search for a new WACC is recommended (p.150). Fernandez (2010) uses an iterative search to reconcile a tax shield valuation with a WACC valuation. Using the simplified ME approach outlined in this paper does not require iterative solutions.

The value of equity in each subsequent period is based on this opening intrinsic value plus the expected levered equity return less any dividend (where dividends include share issues and repurchases). There are therefore different values for equity at the start of each period, including the first accounting period in the terminal period, described as period T+1. Therefore the WACC rate changes over time in response to these different intrinsic equity values, including the T+1 period. Such an approach is sometimes called 'time varying discount rates'. The advantage of using time varying discount rates is that any leverage level can be accommodated, alleviating the need for some of the restrictive leverage assumptions of Miles and Ezzell (1985). No adjustment is made for inflation and all discount rates and accounting numbers are at nominal value, in line with the arguments presented in O'Hanlon and Peasnell (2004). The only discount rates that are direct inputs are the unlevered betas, risk-free rates and market premia. These inputs can be changed over the forecast period but have been kept the same in this paper for simplicity.

(iv) Bifurcation of the Terminal Period

One of the major contributions of this paper is that the calculations shown in appendix 2 demonstrate that the terminal period should be split into two. Two forecast 'periods' are required at the end of the finite forecast period in order to achieve equivalence between the different valuation methods. Although the problems of terminal period valuation have been discussed by Berkman et al (1998), the author believes that this is the first time that this concept has been shown using an example. When forecasting financial variables such as income, equity book values, interest and dividends constant growth in the first period of the terminal period (T+1) is not possible because it would violate the balance sheet equation. Stable growth for all accounting variables only occurs in the second period of the terminal period (T+2). The examples in the appendices show this – interest paid in T+1 is based on the book debt balance at t_4 and therefore cannot grow at the terminal growth rate. In this approach the dividends are the balancing variable in the T+1 period. Any variable can be a balancing figure but dividends are the most logical choice because for any firm to have any value it must eventually pay a dividend (share repurchases, special dividends etc. are included in the definition of dividends). Given that the accounting variables cannot all grow equally in the T+1 period, it is interesting to assert that the formulae in most finance textbooks are incorrect – applying one growth rate to all the accounting variables in the last period of the finite forecasting period results in accounts that do not balance! In appendix one, the author has therefore replaced g in the traditional formulae (1+g) with T+1.

IV. Practical Examples

Three examples are shown in appendix 2. The first example is a perpetuity with no added value where debt and equity intrinsic values equal book values. One important point to note is that the choice of asset beta formula has a large effect on equity value. At the end of the first example, a table is presented which shows the effect of applying the alternative definitions for levering and unlevering the equity betas. The present value of the tax shield under MM in case 1 in appendix 2 would be $120,000 (400,000*30%) whereas under the simplified ME framework it is $75,000. The importance of this decision cannot be underestimated as small changes in discount rates can have large effects on tax shield valuations. Given this uncertainty, the author would argue that other valuation methods are also used so that the APV result can be checked for reasonableness. For example, the analyst can look at the WACCs produced to see if they are reasonable or if the dividend discount model gives a sensible answer e.g. that a perpetuity $60,000 dividend with a 10% Ke gives an equity value of $600,000. It is therefore argued that the simplified ME approach should be used in conjunction with the other standard approaches (DDM, DFCF etc) to ensure a reasonable equity valuation overall.

The second example is similar to the example in Penman (2001), who uses the example to argue that that all equity valuation methods are not equivalent. As can be seen, the answers are equivalent. There are no dividend payments in the finite forecasting period, reducing equity value when compared with example 1, affecting leverage and thus increasing WACC. It can be seen that the present value of the tax shield is the same as example 1 as the firm is making the same profit and the discount rate is the same.

In the third example, the firm makes a loss of 60,000 in t_2 which means that no legal dividend can be paid in t_2 or t_3. In most tax jurisdictions, the loss in t_2 can be carried forward to t_3 meaning that no firm income tax is payable in t_2 or t_3, reducing the benefit of the tax shield and supporting the argument for using K_u as the tax shield discount rate, as the variability of tax shield benefits is varying with operational cash flows. The effective firm income tax rate in t_2 and t_3 is zero, following equation (1) above. The third example also clearly shows how the values ‘trip up’ as they enter the terminal period as a positive 2% terminal growth rate is applied.

V. Conclusion

The paper presents a unique demonstration of the adjusted present value (APV) method of equity valuation. Using theoretical and practical evidence, it argues that the unlevered cost of equity should be used to discount the tax shield in all forecast periods. Using a simple formula from Miles and Ezzell (1985), levered equity betas are calculated and input into four other commonly used equity valuation methods. The equivalence of APV and other equity valuation methods is shown. Some of the problems of APV are presented and the author argues that given such weaknesses, the full suite of valuation methods should be used. The practical issues of constructing sensible forecasts are presented and three examples are used to demonstrate equivalence and the robustness of the unlevered cost of equity. It is argued that this simplified ME framework works with any leverage forecast and is hopefully powerful enough to be used in all business transactions and valuations, particularly where there are leverage issues. As such it should be of interest to investment analysts, financial managers responsible for capital investment and business valuers.

Appendix 1 – Formulae used

Variation attribute	Equity valued directly as P_e	Equity valued indirectly as $P_e = P_f - P_d$ *ie.Value of the equity holders' claim at time 0 = Value of the firm to all investors less value of debtholders' claim*	
	Value of P_e	Value of whole firm P_f	Value of debt P_d
Cash	DDM(equity) $P_e = \sum_{t=1}^{T} \frac{D_t}{(1+r_e)^t} + \frac{D_{T+1}}{(r_e - g)(1+r_e)^T}$	FCF(firm) $P_f = \sum_{t=1}^{T} \frac{C_t}{(1+r_w)^t} + \frac{C_{T+1}}{(r_w - g)(1+r_w)^T}$	FCF(debt) $P_d = \sum_{t=1}^{T} \frac{I_t - \Delta L_t}{(1+r_d)^t} + \frac{I_{T+1} - \Delta L_{T+1}}{(r_d - g)(1+r_d)^T}$
Residual income	RI(equity) $P_e = SE_0 \sum_{t=1}^{T} \frac{RI_t}{(1+r_e)^t} + \frac{RI_{T+1}}{(r_e - g)(1+r_e)^T}$ $where RI_t = NI_t - r_e SE_{t-1}$	RI(firm) $P_f = SE_0 \sum_{t=1}^{T} \frac{ROI_t}{(1+r_w)^t} + \frac{ROI_{T+1}}{(r_w - g)(1+r_w)^T}$ $where ROI_t = OI_t - r_w OA_{t-1}$	RI(debt) $P_d = L_0 \sum_{t=1}^{T} \frac{RID_t}{(1+r_d)^t} + \frac{RID_{T+1}}{(r_d - g)(1+r_d)^T}$ $where RID_t = I_t - r_d L_{t-1}$
Cash		APV (firm) *Pf = CF(firm) + DVTS* $P_f = \left[\sum_{t=1}^{T} \frac{C_t}{(1+r_{eu})^t} + \frac{C^*_{T+1}}{(r_{eu} - g)(1+r_{eu})^T}\right] + \left[\sum_{t=1}^{T} \frac{L_t.EFIT.r_d}{(1+r_{eu})^t} + \frac{DVTS_{T+1}}{(r_{eu} - g)(1+r_{eu})^T}\right]$	FCF(debt) $P_d = \sum_{t=1}^{T} \frac{I_t - \Delta L_t}{(1+r_d)^t} + \frac{I_{T+1} - \Delta L_{T+1}}{(r_d - g)(1+r_d)^T}$

Where the values for the perpetuities are:

$D_{T+1} = NI_{T+1} - SE_{T+1} + SE_T$

$C_{T+1} = OI_{T+1} - OA_{T+1} + OA_T$

$(I_{T+1} - \Delta L_{T+1}) = I_{T+1} - L_{T+1} + L_T$

$RI_{T+1} = NI_{T+1} - r_e SE_T$

$ROI_{T+1} = OI_{T+1} - r_w OA_T$

$RID_{T+1} = I_{T+1} - r_d L_T$

$C^*_{T+1} = NI_{T+1} - (L_T.R_{dECT}(1+g).EFIT) - \Delta OA_{T+1}$

$DVTS_{T+1} = L_T.r_{dT+1}.EFIT_{T+1}$

And:

OA_t = Operating asset balance at time *t*

L_t = Liability balance at time t

SE_t = Shareholders' equity at time t: i.e. $OA_t - L_t = SE_t$

OI_t = Operating income for the period ending at time t, net of tax

I_t = interest expense for the period ending at time t, net of tax

NI_t = Net income for the period ending at time t; $NI_t = OI_t - I_t$

D_t = Dividends paid to common equity; $SE_t = SE_{t-1} + NI_t - D_t$

C_t = Free Cash Flow; $C_t = OI_t - \Delta OA$

Appendix 2 Example 1 – A Perpetuity

FORECAST COSTS OF CAPITAL	t_1	t_2	t_3	t_4	T+1	
R_f = Risk Free Rate	3.00%	3.00%	3.00%	3.00%	3.00%	
R_m-R_f = Market Premium	5.00%	5.00%	5.00%	5.00%	5.00%	
Average debt interest cost	5.00%	5.00%	5.00%	5.00%	5.00%	
β_d = Beta of debt	0.4000	0.4000	0.4000	0.4000	0.4000	
$R_d = R_f + \beta_d(Rf-Rm)$ = Cost of debt before tax	5.00%	5.00%	5.00%	5.00%	5.00%	
FIT = Firm Income Tax rate	30.00%	30.00%	30.00%	30.00%	30.00%	
EFIT = Effective Firm Income Tax rate	30.00%	30.00%	30.00%	30.00%		
β_{eu} = Beta of ungeared equity	1.0000	1.0000	1.0000	1.0000	1.0000	
$R_{eu} = Rf + \beta_{eu}(Rf-Rm)$ = Cost of ungeared equity	8.00%	8.00%	8.00%	8.00%	8.00%	
$(1+R_{eu})^{-t}$	0.9259	0.8573	0.7938	0.7350		
$P_{e\,=}$ (intrinsic value from APV)	600,000	600,000	600,000	600,000	600,000	
P_d = Intrinsic Value of Debt (from FCF to debt calculation)	400,000	400,000	400,000	400,000	400,000	
$P_e + P_d$	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	
β_{eg} = Beta of geared equity $(\beta_{eu}+(\beta_{eu}-\beta_d)(P_d/P_e))$	1.4000	1.4000	1.4000	1.4000	1.4000	
$R_{eg} = Rf + \beta_{eg}(Rf-Rm)$ = Cost of geared equity	10.00%	10.00%	10.00%	10.00%	10.00%	
$(1+r_{eg})^{-t}$	0.9091	0.8264	0.7513	0.6830		
$R_{d.ECT} = r_d$ after effective tax	3.50%	3.50%	3.50%	3.50%	3.50%	
$(1+r_{d.ECT})^{-t}$	0.9662	0.9335	0.9019	0.8714		
$r_w=(R_{eg}*(SE_{t-1}/(SE_{t-1}+L_{t-1})))+(r_{dECT}*(l_{t-1}/(SE_{t-1}+L_{t-1})))$	7.40%	7.40%	7.40%	7.40%	7.40%	
$(1+r_w)^{-t}$	0.9311	0.8669	0.8072	0.7516		

Terminal Period assumptions			TERMINAL VALUES	
Growth in OA	0.00%		$D_{T+1} = NI_{T+1} - SE_{T+1} + SE_T$	60,000
Growth in L	0.00%		$RI_{T+1} = NI_{T+1} - r_e SE_T$	-
Growth in SE	0.00%		$C_{T+1} = OI_{T+1} - OA_{T+1} + OA_T$	74,000
Growth in OI	0.00%		$ROI_{T+1} = OI_{T+1} - r_w OA_T$	0
Growth in I	0.00%		$RIT_{\,T+1} = I_{\,T+1} - r_{\,d} L_{\,T}$	-
Growth in NI	0.00%		$(I_{T+1} - DL_{T+1}) = I_{T+1} - L_{T+1} + L_T$	14,000
Growth in dividends is the balancing figure			$C^*_{T+1} = NI_{T+1} - (L_T.R_{dECT}(1+g).EFIT)$	74,000
			$DVTS_{T+1} = L_{T+1}.r_{dT+1}.EFIT_T$	6,000

SUMMARISED FORECAST FINANCIAL STATEMENTS									
Statements of Position	t_0	t_1	t_2	t_3	t_4	T+1	*% Change*	T+2	*% Change*
OA_t	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	*0.00%*	1,000,000	*0.00%*
L_t	- 400,000	-400,000	-400,000	-400,000	-400,000	-400,000	*0.00%*	-400,000	*0.00%*
$SE_t=OA_t-L_t$	600,000	600,000	600,000	600,000	600,000	600,000	*0.00%*	600,000	*0.00%*
Comprehensive Income Statements									
OI_t after ECT		74,000	74,000	74,000	74,000	74,000	*0.00%*	74,000	*0.00%*
I_t after ECT		-14,000	-14,000	-14,000	-14,000	-14,000	*0.00%*	-14,000	*0.00%*
$NI_t=OI_t-I_t$		60,000	60,000	60,000	60,000	60,000	*0.00%*	60,000	*0.00%*
$-D_t$		- 60,000	- 60,000	- 60,000	- 60,000	-60,000	*0.00%*	-60,000	*0.00%*
$RP_t=NI_t-D_t$		0	0	0	0	0		0	
Statement of Cash Flows									
$C_t=OI_t$-ChangeOA_t		74,000	74,000	74,000	74,000	74,000	*0.00%*	74,000	*0.00%*

Appendix 2 Example 1 (cont) – A Perpetuity

EQUITY VALUATIONS					
Equity Valued Directly as P_e	**t_1**	**t_2**	**t_3**	**t_4**	**Sum**
1. DDM Model - Cash Flows to Equity					
D_t	60,000	60,000	60,000	60,000	
$(1+r_{eg})^{-t}$	0.9091	0.8264	0.7513	0.6830	
$D_t(1+r_{eg})^{-t}$	54,545	49,587	45,079	40,981	190,192
$(D_{T+1})/r_{eg}\text{-}g)\ (1+r_{eg})^{-T}$					409,808
P_e					**600,000**
2. Residual Income Model					
SE_{t-1}	600,000	600,000	600,000	600,000	
r_{eg}	10.00%	10.00%	10.00%	10.00%	
$r_{eg}SE_{t-1}$	60,000	60,000	60,000	60,000	
NI	60,000	60,000	60,000	60,000	
$RI=NI-r_{eg}SE_{t-1}$	0	0	0	0	
$(1+r_{eg})^{-t}$	0.9091	0.8264	0.7513	0.6830	
$RI(1+r_{eg})^{-t}$	0	0	0	0	0
$(RI_{T+1})/r_e\text{-}g)\ (1+r_e)^{-T}$					0
SE_{t-1}					600,000
P_e					**600,000**
Equity Valued Indirectly as $P_e = P_j - P_d$					
3a. Free Cash Flow Model - Cash Flows to the Firm					
C_t	74,000	74,000	74,000	74,000	
$(1+r_w)^{-t}$	0.9311	0.8669	0.8072	0.7516	
$C_t(1+r_w)^{-t}$	68,901	64,154	59,734	55,618	248,407
$(C_{T+1})/r_w\text{-}g)\ (1+r_w)^{-T}$					751,593
P_j					1,000,000
3b. Free Cash Flow to Debt					
I_t	14,000	14,000	14,000	14,000	
ChL_t	0	0	0	0	
$I_t\text{-}ChL_t$	14,000	14,000	14,000	14,000	
$(1+r_d)^{-t}$	0.9662	0.9335	0.9019	0.8714	
$(I_t\text{-}ChL_t)(1+r_d)^{-t}$	13,527	13,069	12,627	12,200	51,423
$((I_{T+1}\text{-}CHL_{T+1})/(r_d\text{-}g))(1+r_d)^{-T}$					348,577
P_d					400,000
$P_e = P_j - P_d$					**600,000**

Appendix 2 Example 1 (cont) – A Perpetuity

4a. Residual Income Model for the Firm					
OA_{t-1}	1,000,000	1,000,000	1,000,000	1,000,000	
r_w	7.40%	7.40%	7.40%	7.40%	
r_wOA_{t-1}	74,000	74,000	74,000	74,000	
OI	74,000	74,000	74,000	74,000	
$ROI=OI-r_wOA_{t-1}$	0	0	0	0	
$(1+r_w)^{-t}$	0.9311	0.8669	0.8072	0.7516	
$ROI(1+r_w)^{-t}$	0	0	0	0	0
$(ROI_{T+1})/r_w\text{-}g)\ (1+r_w)^{-T}$					0
OA_{t-1}					1,000,000
P_j					1,000,000
4b. Residual Income for Debt					
L_{t-1}	400,000	400,000	400,000	400,000	
r_d	3.50%	3.50%	3.50%	3.50%	
r_dL_{t-1}	14,000	14,000	14,000	14,000	
I_t	14,000	14,000	14,000	14,000	
$RID=I-r_dL_{t-1}$	0	0	0	0	
$(1+r_d)^{-t}$	0.966	0.934	0.902	0.871	
$RID(1+r_d)^{-t}$	0	0	0	0	0
$(RID_{T+1})/r_d\text{-}g)\ (1+r_d)^{-T}$					0
L_{t-1}					400,000
P_d					400,000
$P_e = P_j - P_d$					**600,000**
5. APV using a Simplified Miles-Ezzell Framework					
L_{t-1}	400,000	400,000	400,000	400,000	
R_d	5.00%	5.00%	5.00%	5.00%	
$EFIT_t$	30.00%	30.00%	30.00%	30.00%	
$L_{t-1.}R_d.EFIT$	6,000	6,000	6,000	6,000	
$(1+r_{eu})^{-t}$	0.9259	0.8573	0.7938	0.7350	
$L_{t-1.}R_d.EFIT.\ (1+r_{eu})^{-t}$	5,556	5,144	4,763	4,410	19,873
$DVTS/(r_{eu}\text{-}g)$					55,127
P_{ts}					75,000
C_t	74,000	74,000	74,000	74,000	
$(1+r_{eu})^{-t}$	0.9259	0.8573	0.7938	0.7350	
$C_t.(1+r_{eu})^{-t}$	68,519	63,443	58,744	54,392	245,097
$((C^*_{T+1})/(R_{eu}\text{-}g))\ (1+r_{eu})^{-T}$					679,903
P_{eu}					925,000
P_d					400,000
$P_e = P_{ts} + P_{eu} - P_d$					**600,000**

Appendix 2 Example 1 (cont) Comparison of Different Asset Beta Formulae

Using the same financial statement data as above but assuming that the observed levered equity beta is 1.4 and the personal tax rate is 22%, the alternative formulae for unlevering the equity betas generate the following unlevered equity betas:

MM unlevered beta (2)	0.9520
ME unlevered beta (3)	1.0000
Goetzmann unlevered beta (4)	0.8400
Cooper unlevered beta (5)	0.9274
Tham unlevered beta (6)	0.9745

Using the CAPM with the same risk free rate (3%) and market premium (5%) we get the following unlevered equity discount rates:

MM Ku	7.7600%
ME Ku	8.0000%
Goetzmann Ku	7.2000%
Cooper Ku	7.6370%
Tham Ku	7.8725%

Which yield the following equity valuations:

APV Values	**FCF @ Ku**	**PV Tax Shield**	**Intrinsic Equity value**
MM (tax shield at Kd)	953,608	120,000	673,608
ME simplified (tax shield at Ku)	925,000	75,000	600,000
Goetzmann (tax shield at Ku)	1,027,778	83,333	711,111
Cooper (tax shield at Ku)	968,967	78,565	647,532
Tham (tax shield at Ke)	940,000	60,000	600,000

Appendix 2 Example 2 – No Dividend in Finite Forecast Period

FORECAST COSTS OF CAPITAL	t_1	t_2	t_3	t_4	T+1		
R_f = Risk Free Rate	3.00%	3.00%	3.00%	3.00%	3.00%		
R_m-R_f = Market Premium	5.00%	5.00%	5.00%	5.00%	5.00%		
Average debt interest cost	5.00%	5.00%	5.00%	5.00%	5.00%		
β_d = Beta of debt	0.4000	0.4000	0.4000	0.4000	0.4000		
$R_d = R_f + \beta_d(Rf-Rm)$ = Cost of debt before tax	5.00%	5.00%	5.00%	5.00%	5.00%		
FIT = Firm Income Tax rate	30.00%	30.00%	30.00%	30.00%	30.00%		
EFIT = Effective Firm Income Tax rate	30.00%	30.00%	30.00%	30.00%			
β_{eu} = Beta of ungeared equity	1.0000	1.0000	1.0000	1.0000	1.0000		
$R_{eu} = Rf + \beta_{eu}(Rf-Rm)$ = Cost of ungeared equity	8.00%	8.00%	8.00%	8.00%	8.00%		
$(1+R_{eu})^{-t}$	0.9259	0.8573	0.7938	0.7350			
$P_{e\,=}$ (intrinsic value from APV)	401,272	445,374	493,004	544,444	600,000		
P_d = Intrinsic Value of Debt (from FCF to debt calculation)	400,000	400,000	400,000	400,000	400,000		
$P_e + P_d$	801,272	845,374	893,004	944,444	1,000,000		
β_{eg} = Beta of geared equity $(\beta_{eu}+(\beta_{eu}-\beta_d)(P_d/P_e))$	1.5981	1.5389	1.4868	1.4408	1.4000		
$R_{eg} = Rf + \beta_{eg}(Rf-Rm)$ = Cost of geared equity	10.99%	10.69%	10.43%	10.20%	10.00%		
$(1+r_{eg})^{-t}$	0.9010	0.8139	0.7370	0.6688			
$R_{d.ECT} = r_d$ after effective tax	3.50%	3.50%	3.50%	3.50%	3.50%		
$(1+r_{d.ECT})^{-t}$	0.9662	0.9335	0.9019	0.8714			
$r_w=(R_{eg}*(SE_{t-1}/(SE_{t-1}+L_{t-1})))+(r_{dECT}*(l_{t-1}/(SE_{t-1}+L_{t-1})))$	7.25%	7.29%	7.33%	7.36%	7.40%		
$(1+r_w)^{-t}$	0.9324	0.8690	0.8097	0.7542			

Terminal Period assumptions			TERMINAL VALUES	
Growth in OA	0.00%		$D_{T+1} = NI_{T+1} - SE_{T+1} + SE_T$	60,000
Growth in L	0.00%		$RI_{T+1} = NI_{T+1} – r_eSE_T$	- 24,000
Growth in SE	0.00%		$C_{T+1} = OI_{T+1} - OA_{T+1} + OA_T$	74,000
Growth in OI	0.00%		$ROI_{T+1} = OI_{T+1} – r_wOA_T$	- 17,760
Growth in I	0.00%		$RIT_{\,T+1} = I_{\,T+1} – r_{\,d}L_{\,T}$	-
Growth in NI	0.00%		$(I_{T+1} - DL_{T+1}) = I_{T+1} - L_{T+1} + L_T$	14,000
Growth in dividends is the balancing figure			$C^*_{T+1} = NI_{T+1} – (L_T.R_{dECT}(1+g).EFIT)$	74,000
			$DVTS_{T+1} = L_{T+1}.r_{dT+1}.EFIT_T$	6,000

SUMMARISED FORECAST FINANCIAL STATEMENTS									
Statements of Position	t_0	t_1	t_2	t_3	t_4	T+1	*% Change*	T+2	*% Change*
OA_t	1,000,000	1,060,000	1,120,000	1,180,000	1,240,000	1,240,000	*0.00%*	1,240,000	*0.00%*
L_t	- 400,000	-400,000	-400,000	-400,000	-400,000	-400,000	*0.00%*	-400,000	*0.00%*
$SE_t=OA_t-L_t$	600,000	660,000	720,000	780,000	840,000	840,000	*0.00%*	840,000	*0.00%*
Comprehensive Income Statements									
OI_t after ECT		74,000	74,000	74,000	74,000	74,000	*0.00%*	74,000	*0.00%*
I_t after ECT		-14,000	-14,000	-14,000	-14,000	-14,000	*0.00%*	-14,000	*0.00%*
$NI_t=OI_t-I_t$		60,000	60,000	60,000	60,000	60,000	*0.00%*	60,000	*0.00%*
$-D_t$		-	-	-	-	-60,000	*#DIV/0!*	-60,000	*0.00%*
$RP_t=NI_t-D_t$		60,000	60,000	60,000	60,000	0		0	
Statement of Cash Flows									
$C_t=OI_t$-ChangeOA$_t$		14,000	14,000	14,000	14,000	74,000	*428.57%*	74,000	*0.00%*

Appendix 2 Example 2 (cont) – No Dividend in Finite Forecast Period

EQUITY VALUATIONS					
Equity Valued Directly as P_e	**t_1**	**t_2**	**t_3**	**t_4**	**Sum**
1. DDM Model - Cash Flows to Equity					
D_t	0	0	0	0	
$(1+r_{eg})^{-t}$	0.9010	0.8139	0.7370	0.6688	
$D_t(1+r_{eg})^{-t}$	0	0	0	0	0
$(D_{T+1})/r_{eg}-g)\ (1+r_{eg})^{-T}$					401,272
P_e					**401,272**
2. Residual Income Model					
SE_{t-1}	600,000	660,000	720,000	780,000	
r_{eg}	10.99%	10.69%	10.43%	10.20%	
$r_{eg}SE_{t-1}$	65,943	70,583	75,125	79,592	
NI	60,000	60,000	60,000	60,000	
$RI=NI-r_{eg}SE_{t-1}$	-5,943	-10,583	-15,125	-19,592	
$(1+r_{eg})^{-t}$	0.9010	0.8139	0.7370	0.6688	
$RI(1+r_{eg})^{-t}$	-5,354	-8,614	-11,148	-13,103	-38,219
$(RI_{T+1})/r_e-g)\ (1+r_e)^{-T}$					-160,509
SE_{t-1}					600,000
P_e					**401,272**
Equity Valued Indirectly as $P_e = P_j - P_d$					
3a. Free Cash Flow Model - Cash Flows to the Firm					
C_t	14,000	14,000	14,000	14,000	
$(1+r_w)^{-t}$	0.9324	0.8690	0.8097	0.7542	
$C_t(1+r_w)^{-t}$	13,053	12,166	11,336	10,558	47,114
$(C_{T+1})/r_w-g)\ (1+r_w)^{-T}$					754,158
P_j					801,272
3b. Free Cash Flow to Debt					
I_t	14,000	14,000	14,000	14,000	
ChL_t	0	0	0	0	
I_t-ChL_t	14,000	14,000	14,000	14,000	
$(1+r_d)^{-t}$	0.9662	0.9335	0.9019	0.8714	
$(I_t-ChL_t)(1+r_d)^{-t}$	13,527	13,069	12,627	12,200	51,423
$((I_{T+1}-CHL_{T+1})/(r_d-g))(1+r_d)^{-T}$					348,577
P_d					400,000
$P_e = P_j - P_d$					**401,272**

Appendix 2 Example 2 (cont) – No Dividend in Finite Forecast Period

4a. Residual Income Model for the Firm					
OA_{t-1}	1,000,000	1,060,000	1,120,000	1,180,000	
r_w	7.25%	7.29%	7.33%	7.36%	
r_wOA_{t-1}	72,512	77,277	82,075	86,904	
OI	74,000	74,000	74,000	74,000	
$ROI=OI-r_wOA_{t-1}$	1,488	-3,277	-8,075	-12,904	
$(1+r_w)^{-t}$	0.9324	0.8690	0.8097	0.7542	
$ROI(1+r_w)^{-t}$	1,387	-2,848	-6,538	-9,731	-17,730
$(ROI_{T+1})/r_w\text{-}g)\ (1+r_w)^{-T}$					-180,998
OA_{t-1}					1,000,000
P_j					801,272
4b. Residual Income for Debt					
L_{t-1}	400,000	400,000	400,000	400,000	
r_d	3.50%	3.50%	3.50%	3.50%	
r_dL_{t-1}	14,000	14,000	14,000	14,000	
I_t	14,000	14,000	14,000	14,000	
$RID=I-r_dL_{t-1}$	0	0	0	0	
$(1+r_d)^{-t}$	0.966	0.934	0.902	0.871	
$RID(1+r_d)^{-t}$	0	0	0	0	0
$(RID_{T+1})/r_d\text{-}g)\ (1+r_d)^{-T}$					0
L_{t-1}					400,000
P_d					400,000
$P_e = P_j - P_d$					**401,272**
5. APV using a Simplified Miles-Ezzell Framework					
L_{t-1}	400,000	400,000	400,000	400,000	
R_d	5.00%	5.00%	5.00%	5.00%	
$EFIT_t$	30.00%	30.00%	30.00%	30.00%	
$L_{t-1.}R_d.EFIT$	6,000	6,000	6,000	6,000	
$(1+r_{eu})^{-t}$	0.9259	0.8573	0.7938	0.7350	
$L_{t-1.}R_d.EFIT.\ (1+r_{eu})^{-t}$	5,556	5,144	4,763	4,410	19,873
$DVTS/(r_{eu}\text{-}g)$					55,127
P_{ts}					75,000
C_t	14,000	14,000	14,000	14,000	
$(1+r_{eu})^{-t}$	0.9259	0.8573	0.7938	0.7350	
$C_t.(1+r_{eu})^{-t}$	12,963	12,003	11,114	10,290	46,370
$((C^*_{T+1})/(R_{eu}\text{-}g))\ (1+r_{eu})^{-T}$					679,903
P_{eu}					726,272
P_d					400,000
$P_e = P_{ts} + P_{eu} - P_d$					**401,272**

Appendix 2 Example 3 – Zero Effective Firm Income Tax and Terminal Growth Assumption

FORECAST COSTS OF CAPITAL	t_1	t_2	t_3	t_4	T+1	
R_f = Risk Free Rate	3.00%	3.00%	3.00%	3.00%	3.00%	
R_m-R_f = Market Premium	5.00%	5.00%	5.00%	5.00%	5.00%	
Average debt interest cost	5.00%	5.00%	5.00%	5.00%	5.00%	
β_d = Beta of debt	0.4000	0.4000	0.4000	0.4000	0.4000	
$R_d = R_f + \beta_d(Rf\text{-}Rm)$ = Cost of debt before tax	5.00%	5.00%	5.00%	5.00%	5.00%	
FIT = Firm Income Tax rate	30.00%	30.00%	30.00%	30.00%	30.00%	
EFIT = Effective Firm Income Tax rate	30.00%	0.00%	0.00%	30.00%		
β_{eu} = Beta of ungeared equity	1.0000	1.0000	1.0000	1.0000	1.0000	
$R_{eu} = Rf + \beta_{eu}(Rf\text{-}Rm)$ = Cost of ungeared equity	8.00%	8.00%	8.00%	8.00%	8.00%	
$(1+R_{eu})^{-t}$	0.9259	0.8573	0.7938	0.7350		
$P_{e\,=}$ (intrinsic value from APV)	515,630	508,881	561,591	618,519	620,000	
P_d = Intrinsic Value of Debt (from FCF to debt calculation)	400,000	400,000	400,000	400,000	400,000	
$P_e + P_d$	915,630	908,881	961,591	1,018,519	1,020,000	
β_{eg} = Beta of geared equity $(\beta_{eu}+(\beta_{eu}\text{-}\beta_d)(P_d/P_e))$	1.4654	1.4716	1.4274	1.3880	1.3871	
$R_{eg} = Rf + \beta_{eg}(Rf\text{-}Rm)$ = Cost of geared equity	10.33%	10.36%	10.14%	9.94%	9.94%	
$(1+r_{eg})^{-t}$	0.9064	0.8213	0.7457	0.6783		
$R_{d.ECT} = r_d$ after effective tax	3.50%	5.00%	5.00%	3.50%	3.50%	
$(1+r_{d.ECT})^{-t}$	0.9662	0.9202	0.8764	0.8467		
$r_w=(R_{eg}*(SE_{t-1}/(SE_{t-1}+L_{t-1})))+(r_{dECT}*(l_{t-1}/(SE_{t-1}+L_{t-1})))$	7.34%	8.00%	8.00%	7.41%	7.41%	
$(1+r_w)^{-t}$	0.9316	0.8626	0.7987	0.7436		

Terminal Period assumptions			TERMINAL VALUES	
Growth in OA	2.00%		$D_{T+1} = NI_{T+1} - SE_{T+1} + SE_T$	49,200
Growth in L	2.00%		$RI_{T+1} = NI_{T+1} - r_eSE_T$	1,587
Growth in SE	2.00%		$C_{T+1} = OI_{T+1} - OA_{T+1} + OA_T$	55,200
Growth in OI	2.00%		$ROI_{T+1} = OI_{T+1} - r_wOA_T$	1,082
Growth in I	2.00%		$RIT_{\ T+1} = I_{\ T+1} - r_{\ d}L_{\ T}$	-
Growth in NI	2.00%		$(I_{T+1} - DL_{T+1}) = I_{T+1} - L_{T+1} + L_T$	6,000
Growth in dividends is the balancing figure			$C*_{T+1} = NI_{T+1} - (L_T.R_{dECT}(1+g).EFIT)$	55,080
			$DVTS_{T+1} = L_{T+1}.r_{dT+1}.EFIT_T$	6,120

SUMMARISED FORECAST FINANCIAL STATEMENTS									
Statements of Position	t_0	t_1	t_2	t_3	t_4	T+1	*% Change*	T+2	*% Change*
OA_t	1,000,000	1,000,000	940,000	1,000,000	1,000,000	1,020,000	*2.00%*	1,040,400	*2.00%*
L_t	- 400,000	-400,000	-400,000	-400,000	-400,000	-408,000	*2.00%*	-416,160	*2.00%*
$SE_t=OA_t-L_t$	600,000	600,000	540,000	600,000	600,000	612,000	*2.00%*	624,240	*2.00%*
Comprehensive Income Statements									
OI_t after ECT		74,000	-40,000	80,000	74,000	75,200	*1.62%*	76,704	*2.00%*
I_t after ECT		-14,000	-20,000	-20,000	-14,000	-14,000	*0.00%*	-14,280	*2.00%*
$NI_t=OI_t-I_t$		60,000	- 60,000	60,000	60,000	61,200	*2.00%*	62,424	*2.00%*
$-D_t$		- 60,000	-	-	- 60,000	-49,200	*-18.00%*	-50,184	*2.00%*
$RP_t=NI_t-D_t$		0	-60,000	60,000	0	12,000		12,240	
Statement of Cash Flows									
$C_t=OI_t\text{-}ChangeOA_t$		74,000	20,000	20,000	74,000	55,200	*-25.41%*	56,304	*2.00%*

Appendix 2 Example 3 (cont) – Zero Effective Firm Income Tax and Terminal Growth Assumption

EQUITY VALUATIONS					
Equity Valued Directly as P_e	**t_1**	**t_2**	**t_3**	**t_4**	**Sum**
1. DDM Model - Cash Flows to Equity					
D_t	60,000	0	0	60,000	
$(1+r_{eg})^{-t}$	0.9064	0.8213	0.7457	0.6783	
$D_t(1+r_{eg})^{-t}$	54,384	0	0	40,698	95,082
$(D_{T+1})/r_{eg}$-g) $(1+r_{eg})^{-T}$					420,548
P_e					**515,630**
2. Residual Income Model					
SE_{t-1}	600,000	600,000	540,000	600,000	
r_{eg}	10.33%	10.36%	10.14%	9.94%	
$r_{eg}SE_{t-1}$	61,963	62,149	54,739	59,641	
NI	60,000	-60,000	60,000	60,000	
$RI=NI-r_{eg}SE_{t-1}$	-1,963	-122,149	5,261	359	
$(1+r_{eg})^{-t}$	0.9064	0.8213	0.7457	0.6783	
$RI(1+r_{eg})^{-t}$	-1,780	-100,323	3,924	244	-97,936
$(RI_{T+1})/r_e$-g) $(1+r_e)^{-T}$					13,566
SE_{t-1}					600,000
P_e					**515,630**
Equity Valued Indirectly as $P_e = P_j - P_d$					
3a. Free Cash Flow Model - Cash Flows to the Firm					
C_t	74,000	20,000	20,000	74,000	
$(1+r_w)^{-t}$	0.9316	0.8626	0.7987	0.7436	
$C_t(1+r_w)^{-t}$	68,937	17,251	15,974	55,024	157,186
$(C_{T+1})/r_w$-g) $(1+r_w)^{-T}$					758,444
P_j					915,630
3b. Free Cash Flow to Debt					
I_t	14,000	20,000	20,000	14,000	
ChL_t	0	0	0	0	
I_t-ChL_t	14,000	20,000	20,000	14,000	
$(1+r_d)^{-t}$	0.9662	0.9202	0.8764	0.8467	
$(I_t-ChL_t)(1+r_d)^{-t}$	13,527	18,403	17,527	11,854	61,311
$((I_{T+1}-CHL_{T+1})/(r_d-g))(1+r_d)^{-T}$					338,689
P_d					400,000
$P_e = P_j - P_d$					**515,630**

Appendix 2 Example 3 (cont) – Zero Effective Firm Income Tax and Terminal Growth Assumption

4a. Residual Income Model for the Firm					
OA_{t-1}	1,000,000	1,000,000	940,000	1,000,000	
r_w	7.34%	8.00%	8.00%	7.41%	
$r_w OA_{t-1}$	73,447	80,000	75,200	74,109	
OI	74,000	-40,000	80,000	74,000	
$ROI=OI-r_w OA_{t-1}$	553	-120,000	4,800	-109	
$(1+r_w)^{-t}$	0.9316	0.8626	0.7987	0.7436	
$ROI(1+r_w)^{-t}$	515	-103,509	3,834	-81	-99,241
$(ROI_{T+1})/r_w\text{-}g)\ (1+r_w)^{-T}$					14,871
OA_{t-1}					1,000,000
P_j					915,630
4b. Residual Income for Debt					
L_{t-1}	400,000	400,000	400,000	400,000	
r_d	3.50%	5.00%	5.00%	3.50%	
$r_d L_{t-1}$	14,000	20,000	20,000	14,000	
I_t	14,000	20,000	20,000	14,000	
$RID=I-r_d L_{t-1}$	0	0	0	0	
$(1+r_d)^{-t}$	0.966	0.920	0.876	0.847	
$RID(1+r_d)^{-t}$	0	0	0	0	0
$(RID_{T+1})/r_d\text{-}g)\ (1+r_d)^{-T}$					0
L_{t-1}					400,000
P_d					400,000
$P_e = P_j - P_d$					**515,630**
5. APV using a Simplified Miles-Ezzell Framework					
L_{t-1}	400,000	400,000	400,000	400,000	
R_d	5.00%	5.00%	5.00%	5.00%	
$EFIT_t$	30.00%	0.00%	0.00%	30.00%	
$L_{t-1.}R_d.EFIT$	6,000	0	0	6,000	
$(1+r_{eu})^{-t}$	0.9259	0.8573	0.7938	0.7350	
$L_{t-1.}R_d.EFIT.\ (1+r_{eu})^{-t}$	5,556	0	0	4,410	9,966
$DVTS/(r_{eu}\text{-}g)$					74,973
P_{ts}					84,939
C_t	74,000	20,000	20,000	74,000	
$(1+r_{eu})^{-t}$	0.9259	0.8573	0.7938	0.7350	
$C_t.(1+r_{eu})^{-t}$	68,519	17,147	15,877	54,392	155,934
$((C^*_{T+1})/(R_{eu}\text{-}g))\ (1+r_{eu})^{-T}$					674,757
P_{eu}					830,692
P_d					400,000
$P_e = P_{ts} + P_{eu} - P_d$					**515,630**

References

Arnold, G. (2005). *Corporate Financial Management* third edition FT Prentice Hall.

Arnold, J.A. and Moizer, P. (1984) “A Survey of the Methods Used by UK Investment Analysts to Appraise Investments in Ordinary Shares”, *Accounting and Business Research* (Summer), pp.113-124.

Ashton, D. and Atkins, D. (1978) “Interactions in Corporate Financing and Investment Decisions – Implications for Capital Budgeting: A Further Comment” *The Journal of Finance*, Vol XXXIII, No. 5 pp. 1447-1453

Arzac, E.R. (1996) “Valuation of Highly Leveraged Firms”, Financial Analysts Journal, July/August pp.42-50

Arzac, E.R and L.R. Glosten, (2005) “A Reconsideration of Tax Shield Valuation,” *European Financial Management* 11/4, pp. 453-461.

Barker, R. (1999) “Survey and Market-Based Evidence of Industry-Dependence in Analysts’ Preferences between the Dividend Yield and Price-earnings ratio Valuation Models”, *Journal of Business, Finance and Accounting*, 26 (3/4): pp.393-418.

Barker, R. (2001) *Determining Value*, FT Prentice Hall.

Berkman, H., Bradbury, M.E., Ferguson, J. (1998) “The Magic of Earnings in Terminal Value Calculations”, *Journal of Financial Statement Analysis*, Vol. 3, Summer, Issue 4.

Booth, L., (2002). “Finding Value Where None Exists: Pitfalls in Using Adjusted Present Value,” *Journal of Applied Corporate Finance* 15/1, pp. 8-17.

Bowman, R G., (1979) "The Theoretical Relationship Between Systematic Risk and Financial (Accounting) Risk," *Journal of Finance,* Vol. 34, pp. 617-630.

Brealey, R.A., and Myers, S.C., (2006). *Principles of Corporate Finance*. McGraw-Hill, NewYork.

Cooper, I.A. and Nyborg, K.G., (2004) “*Discount rates and tax*”, unpublished working paper, London Business School, London, UK.

Cooper, I.A., Nyborg, K.G., (2006) “The value of tax shield IS equal to the present value of tax shield”, *Journal of Financial Economics,* 81, 215-225.

Cooper, I.A., Nyborg, K.G., (2008) “Tax Adjusted discount rates with investor taxes and risky debt”, *Financial Management,* Summer pp 365-379.

Damodaran, A., (1994) *Damodaran on Valuation*, John Wiley and Sons, New York.

Dechow, P.M., Hutton, A.P. and Sloan, R.G. (1999) "An Empirical Assessment of the Residual Income Valuation Model", *Journal of Accounting and Economics* 26 pp. 1-34.

Demirakos, E. G., Strong, N.C. and Walker, M. (2004) "What Valuation Models Do Analysts Use?" *Accounting Horizons*, Vol.18, No.4 December pp.221-240.

Demirakos, E. G., Strong, N.C. and Walker, M. (2010) "Does Valuation Model Choice Affect Target price Accuracy?" *European Accounting Review* Vol.19, No.1, pp 35-72.

Ehrhardt, M.C. and Daves, P. R. (2002) "Corporate Valuation: The Combined Impact of Growth and the Tax Shield of Debt on the Cost of Capital and Systematic Risk", *Journal of Applied Finance*, Fall/Winter pp.31-38.

Engel, E., Erikson, M. and Maydew, E. (1999) "Debt Equity Hybrid Securities" *Journal of Accounting Research*, 37, pp.249-273.

Esty, B.C., (1999) "Improved Techniques for Valuing Large-Scale Projects", *Journal of Project Finance*, Spring 1999, 9-25.

Fama, E.F. and French, K.R. (1998) "Taxes, Financing Decisions and Firm Value", *The Journal of Finance*, Vol.53, pp.819-843.

Fernandez, P (2002) *Valuation Methods and Shareholder Value Creation*, Academic Press.

Fernandez, P., (2004) "The Value of Tax shields is NOT equal to the present value of tax shields" *Journal of Financial Economics* 73, 145-165.

Fernandez, P., (2005) "Reply to "Comment on the Value of Tax shields IS equal to the present value of tax shields"" *Quarterly Review of Economics and Finance* 45 pp.188-192.

Fernandez, P. (2010) "*WACC; definition, misconceptions and errors*", SSRN Research Network.

Gamba, A., Sick, G.A. and Leon, C.A. (2008) "Investment Under Uncertainty, Debt and Taxes", *Economic Notes*, Vol.37, No1-2008, pp.31-58.

Goetzmann, W (no date) "An Introduction to Investment Theory", Yale lecture notes,available from: viking.som.yale.edu/will/finman540/classnotes/class7.html (accessed 27.9.2010).

Hamada, R.S., (1972) "The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks," *Journal of Finance*, (May), 435-452.

Harris, R.S., and Pringle, J.J., (1985) "Risk-Adjusted Discount Rates—Extensions from the Average-Risk Case," *Journal of Financial Research*, (Fall), 237-244.

Imam, S., Barker, R. and Clubb, C. (2008) “The Use of Valuation Models by UK Investment Analysts”, *European Accounting Review*, Vol. 17, No.3 pp.503-535

Inselbag, I., and Kaufold, H. (1997) “Two DCF Approaches for Valuing Companies Under Alternative Financing Strategies (And How to Choose Between Them),” *Journal of Applied Corporate Finance*, (Vol. 10, No. 1, Spring), 114-122.

Kaplan, S. N. and Ruback, R.S. (1995) “The Valuation of Cash Flow Forecasts: An Empirical Analysis”, *The Journal of Finance*, Vol. L, No. 4, September pp. 1059-1093

Kemsley, D. and Nissim, D. (2002) “Valuation of the tax Shield”, *The Journal of Finance*, Vol. LVII, No.5, pp.2045-2073.

Kolari, James W. (2010) *“On the Debt Tax Shield Controversy in Corporate Valuation: Discounting at the Levered Cost of Equity”* http://www.mfa-2010.com/papers/Debt%20Tax%20Gains%20Kolari%20MWFA.pdf

Koller, T., Goedhart, M. and Wessels, D. (2010) *Valuation – Measuring and Managing the Value of Companies*, fifth edition, McKinsey & Company , John Wiley and Sons, Inc

Lewellen, W.G., Long, M.S. and McConnell, J.J. (1977) “Capital Cost and Adjusted Present Value: Reconciliation and Amplification’, *Journal of Business Research*, Vol.5, June, pp.109-127

Luehrman, T., (1997) “Using APV: a Better Tool for Valuing Operations,” *Harvard Business Review*, 75, pp. 145–154.

Lundholm, R. and O'Keefe, T. (2001a) “Reconciling Value Estimates from the Discounted Cash Flow Model and the Residual Income Model”, *Contemporary Accounting Research*, Vol 18. No.2. pp.311-335.

Lundholm, R. and O'Keefe, T. (2001b) “On Comparing Residual Income and Discounted Cash Flow Models of Equity Valuation; A Response to Penman 2001 (CAR, Winter 2001)”, *Contemporary Accounting Research*, Winter 2001 Vol. 18 No.4 pp. 693-696.

Massari, M.., Roncaglio, F. and Zanetti, L. (2007) “On the Equivalence Between the APV and the WACC Approach in a Growing Leveraged Firm”, *European Financial Management*, Vol.14, No.1 pp. 152-162

Miles, J. and Ezzell, J. (1980) “The Weighted Average Cost of Capital, Perfect Capital Markets and Project Life: A Clarification”, *Journal of Financial and Quantitative Analysis*, Volume XV, No.3, September, pp.719-730.

Miles, J. and Ezzell, R. (1985) “Reforming Tax Shield Valuation: A Note”, *Journal of Finance*, Vol.XL, pp.1485-1492

Miller, M. H., (1977) “Debt and Taxes”, *Journal of Finance*, Vol. 32, pp. 261-275.

Modigliani, F. and Miller M (1963) “Corporate Income Taxes and the Cost of Capital: A Correction”, *American Economic Review*, (June), pp.433-443.

Myers, S. C (1974) “Interactions of corporate Financing and Investment Decisions – Implications for Capital Budgeting”, *Journal of Finance*, Vol. XXIX No.1 March, pp.1-25.

Oded, J and Michel, A. (2007) “Reconciling DCF Valuation Methodologies”, *Journal of Applied Finance*, Fall/Winter pp.21-32

O’Hanlon, J. and Peasnell, K. (2004) “Residual Income Valuation: Are Inflation Adjustments Necessary?” *Review of Accounting Studies*, 9, pp.375-398

Penman, S. H., and Sougiannis, T. (1998) “A Comparison of Dividend, Cash Flow, and Earnings Approaches to Equity Valuation”, *Contemporary Accounting Research*, 14 pp.343-384.

Penman, S.H. (2001) “On Comparing Cash Flow and Accrual Accounting Models for Use in Equity Valuation: A Response to Lundholm and O’Keefe (CAR, Summer 2001)”, *Contemporary Accounting Research*, Winter 2001 Vol.18 No.4 pp.681-692.

Penman, S.H. (2004) *Financial Statement Analysis and Security Valuation*, Irwin McGraw Hill

Pike, R, Meerjanssen, J. and Chadwick, L. (1993) “The Appraisal of Ordinary Shares by Investment Analysts in the UK and Germany”, *Accounting and Business Research*, Vol 23 No. 92 pp.489-499.

Qi, H. (2010) "Valuation Methodologies and Emerging Markets," *Journal of Business Valuation and Economic Loss Analysis*: Vol. 5 : Iss. 1, Article 2 pp.1-16

Ruback, R., (1995) “A Note on Capital Cash Flow Valuation,” Harvard Business School Case No. 9-295- 069.

Ruback, R., (2002) “Capital Cash Flows: A Simple Approach to Valuing Risky Cash Flows,” *Financial Management* 31, pp. 85–103.

Sabal J. (2007) WACC or APV? *Journal of Business valuation and Economic Loss Analysis* Vol2. Iss.2 pp.1-26

Sharpe, W. (1963) "A Simplified Model for Portfolio Analysis," Management Science (January) pp.277-93.

Shrieves, R.E. and Wachowicz, J. M. (2001) "Free Cash Flow (FCF), Economic Value Added (EVA™), And Net Present Value (NPV): A Reconciliation of Variations of Discounted Cash Flow (DCF) Valuation," *The Engineering Economist:* Volume 46, No.1 pp. 33- 52

Soffer, L. and Soffer, R. (2003) *Financial Statement Analysis*, Prentice Hall.

Stewart, G.B., (1991) *The Quest for Value: the EVA® Management Guide* New York, Harper Collins Publishers, Inc.

Sweeney, R (2002) "Accrual-Accounting Versus Cash-Flow Valuations", working paper McDonough School of Business, Georgetown University.

Taggart, R. A. (1977), "A Model of Corporate Financing Decisions", *Journal of Finance,* V32, pp. 1467 - 1484.

Tham, J. and Velez-Pareja, I. (2001) *The Correct Discount Rate for the tax Shield: the N-period Case*, SSRN Research Network

Tham, J. and Velez-Pareja, I. (2010) *Cost of Capital with Levered Cost of Equity as the Risk of tax Shields,* SSRN Research Network

Velez-Pareja, I. (2008) "*A step by Step Guide to Construct a Financial Model without Plugs and Without Circularity for Valuation Purposes*", SSRN Research Network.

Vélez-Pareja, I. (2010) *Risky Tax Shields and Risky Debt: A Monte Carlo Approach* (June 27, 2010). SSRN Research Network

Wonder, N. (2005) "comment on the value of tax shields is NOT equal to the present value of tax shields', *The Quarterly Review of Economics and Finance*, 45, pp.184-187.