Uncovered Interest Rate Parity: Is the Econometrics Telling us the Wrong Thing?

Keith Pilbeam, City University London

Blog at www.BusinessEconomics.com

Basics of Uncovered Interest Rate Parity Tests

$$F_t = \frac{(1 + r_t^*)S_t}{(1 + r_t)} \quad (1)$$

$$f_t - s_t = r_t^* - r_t \quad (2)$$

$$E[S_{t+1}] = F_t \quad (3)$$

$$E[s_{t+1}] - s_t = r_t^* - r_t \quad (4)$$

$$s_{t+1} = E[s_{t+1}] + \varepsilon_{t+1} \quad (5)$$

$$s_{t+1} - s_t = (f_t - s_t) + \varepsilon_{t+1} \quad (6)$$

$$\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \varepsilon_{t+1} \quad (7)$$

Table 1. Estimates Equation $(s_{t+1} - s_t) = \alpha + \beta(f_t - s_t) + u_{t+1}$

Currency pair	α	β	R2	JB	SC-LM	ARCH
dollar/swiss franc	-0.01	-1.13	0.019	2.66	1.49	1.23
	(0.05)	(0.12)		(0.26)	(0.21)	(0.30)
dollar/yen	-0.02	-1.93	0.048	13.3	2.13	0.88
	(0.007)	(0.01)		(0.001)	(0.10)	(0.45)
dollar/euro	-0.006	-0.55	0.004	0.44	2.26	0.38
	(0.31)	(0.47)		(0.80)	(0.08)	(0.76)
dollar/pound	0.009	-1.42	0.03	5.16	3.40	0.25
	(0.11)	(0.05)		(0.07)	(0.02)	(0.85)

Notes: p-values are in parentheses. JB is the Jarque-Bera test for normality of the residuals, SC-LM is the Breusch-Godfrey lagrange multiplier F-statistic for serial correlation –the alternative hypothesis has three lags and ARCH is the Arch F-statistic for conditional heteroscedasticity – the alternative hypothesis has three lags.

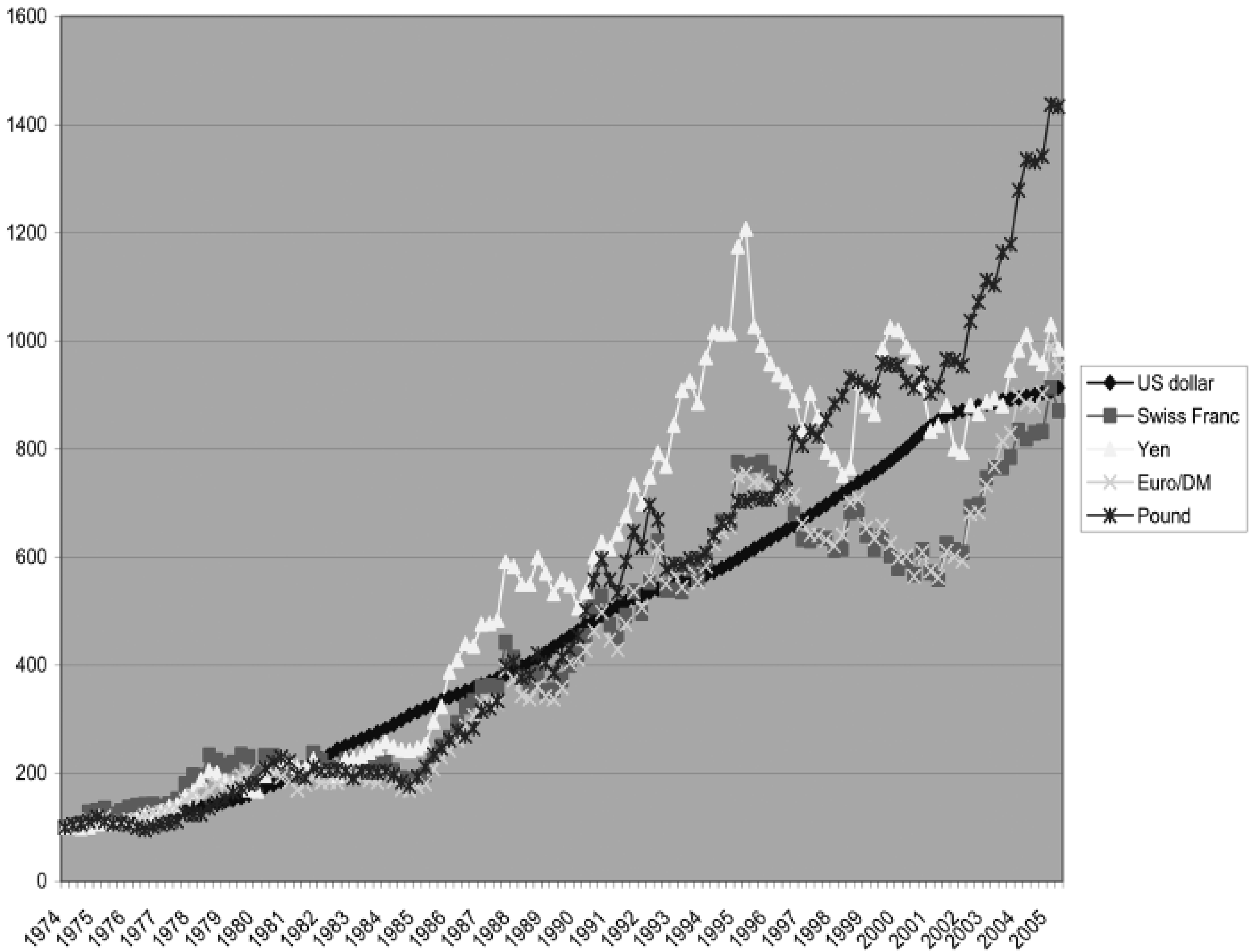
Supposed failure of Uncovered Interest Parity condition then led to a huge literature to review the causes of the failure:

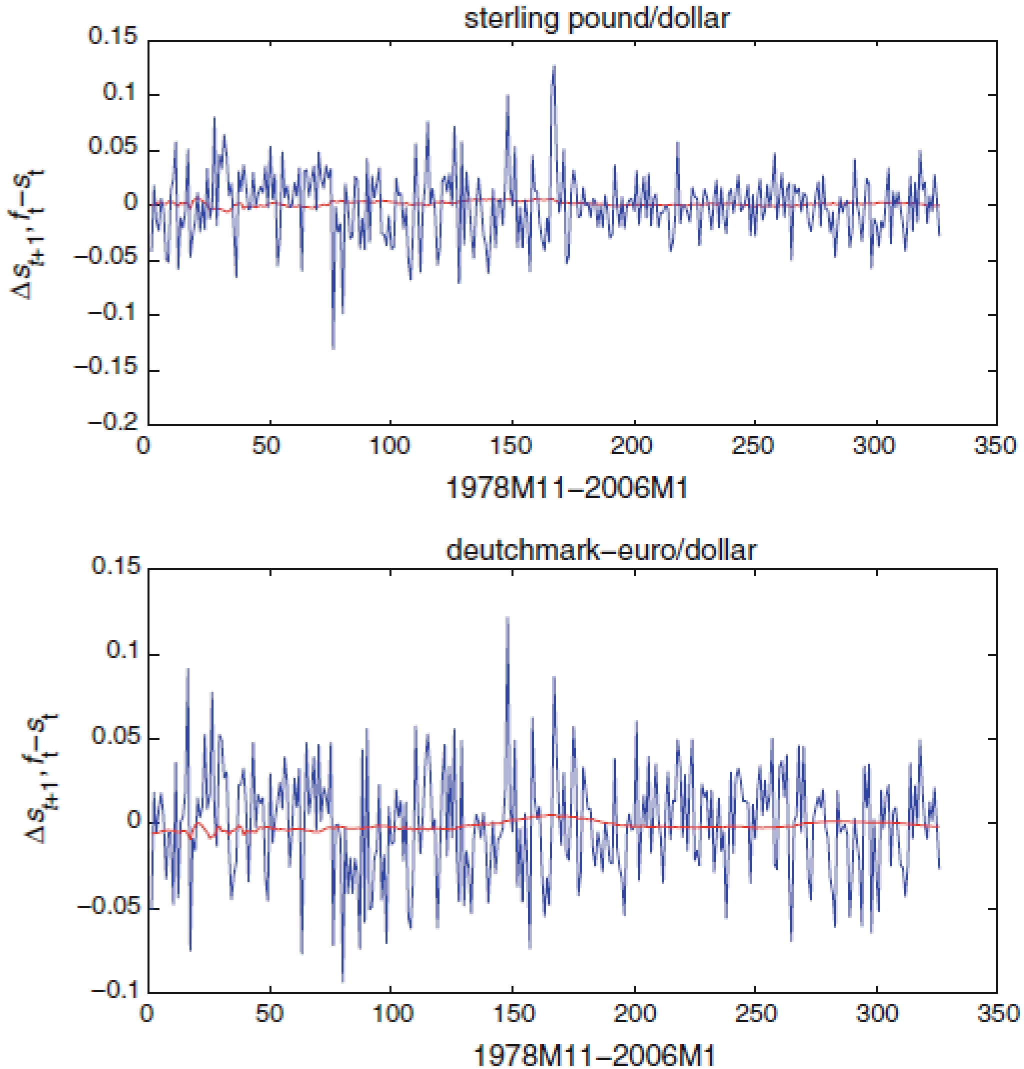
1) A risk premium possibly time varying – What explains the risk premium and how can we model it ?

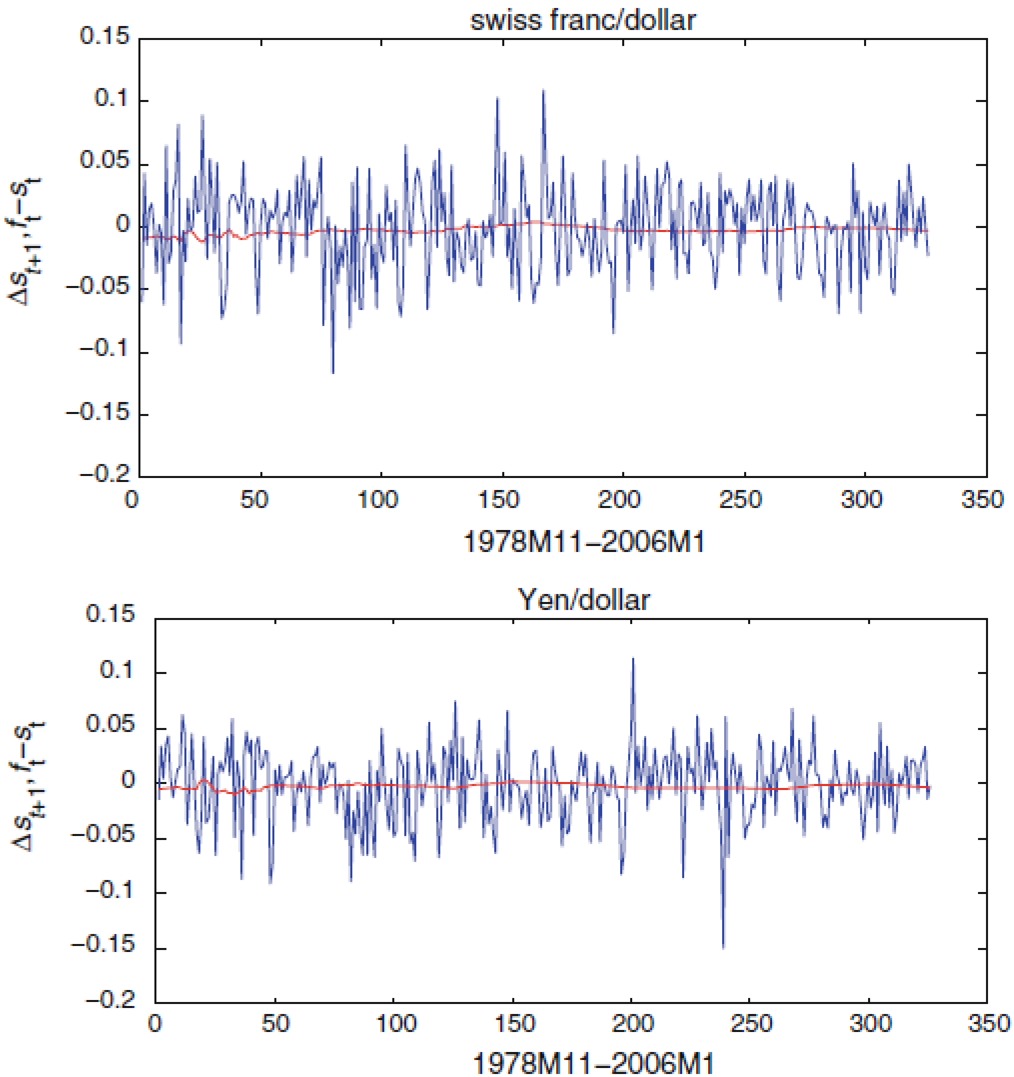
2) Market irrationality a failure of market participants to ensure UIP holds. How do we model such irrationality?

3) Some combination of the above two – but then how do we split the failure between the two?

Figure A2. The Cumulative Dollar Sum of Keeping Capital in US Dollars or the Foreign Currency.




sterling pound/dollar
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
Δs_{t+1}, f_t−s_t
0
50
100
150
200
250
300
350
1978M11–2006M1
deutchmark–euro/dollar
0.15
0.1
0.05
0
−0.05
−0.1
Δs_{t+1}, f_t−s_t
0
50
100
150
200
250
300
350
1978M11–2006M1


swiss franc/dollar
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
0
50
100
150
200
250
300
350
1978M11–2006M1
Yen/dollar
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
0
50
100
150
200
250
300
350
1978M11–2006M1

Thus, taking the nth order Taylor approximation [4] of s_{t+1} about $E[S_{t+1}]$, we have:

$$s_{t+1} = \ln E[S_{t+1}] + \frac{1}{E[S_{t+1}]}(S_{t+1} - E[S_{t+1}]) - \frac{1}{2E^2[S_{t+1}]}(S_{t+1} - E[S_{t+1}])^2$$
$$+\frac{1}{3E^3[S_{t+1}]}(S_{t+1} - E[S_{t+1}])^3 + \cdots + \frac{1}{nE^n[S_{t+1}]}(S_{t+1} - E[S_{t+1}])^n$$
$$+R_{n+1}(S_{t+1}) \tag{11}$$

with $R_{n+1}(.)$ the remainder term of order $n + 1$.

By Taylor's formula, the conditional expected value of the log of the exchange rate obtained from truncating expression (11) for $n=2$ is as follows:

$$E[s_{t+1}] = \ln E[S_{t+1}] - \frac{1}{2E^2[S_{t+1}]}Var(S_{t+1}) + R_3(S_{t+1}) \tag{12}$$

Assume that the remainder term R_3 in equation (12) takes the Lagrange form, then substracting s_t on both sides of (11) it is simple to see that if the foreign exchange market is efficient then the difference of the log exchange rate is given by the following:

$$\Delta s_{t+1} = (f_t - s_t) + \left[\left(\frac{S_{t+1}}{F_t}\right) - 1\right] - \frac{1}{2}\left[\left(\frac{S_{t+1}}{F_t}\right) - 1\right]^2 + R_3\left(\frac{S_{t+1}}{F_t}\right) \quad (13)$$

Applying the expectations operator to equation (13) the expected rate of depreciation is:

$$E[\Delta s_{t+1}] = (f_t - s_t) - \frac{1}{2} Var\left[\frac{S_{t+1}}{F_t}\right] + E\left[R_3\left(\frac{S_{t+1}}{F_t}\right)\right] \quad (14)$$

where $Var(S_{t+1}/F_t)$ is the conditional variance and the expected value of R_3 is the *skewness* of the ratio S_{t+1}/F_t.To simplify this equation we can further assume that the conditional distribution (note throughout that the distribution of interest is conditional on the set of information available at time t) of S_{t+1}/F_t is symmetric, and hence $Skew[S_{t+1}/F_t] = 0$, yielding:

$$E[\Delta s_{t+1}] = (f_t - s_t) - \frac{1}{2} Var\left[\frac{S_{t+1}}{F_t}\right] \quad (15)$$

We also obtain from equation (2) the following:

$$E[\Delta s_{t+1}] = (r_t^* - r_t) - \frac{1}{2} Var\left[\frac{S_{t+1}}{F_t}\right] \quad (16)$$

A New Econometric Test of Uncovered Interest Parity

we can assume $E[S_{t+1}] = F_t + A$, where $A \neq 0$ is a constant risk premium,

$$E[s_{t+1}] = \ln(F_t + A) - \frac{1}{2E[S_{t+1}]^2} Var(S_{t+1}) \tag{21}$$

If the deviation A from efficiency is small compared to the value of the forward contract the expected rate of depreciation can be expressed as:

$$E[\Delta s_{t+1}] = (f_t - s_t) + \frac{A}{F_t} - \frac{1}{2} Var\left(\frac{S_{t+1}}{F_t + A}\right) \tag{22}$$

given that $\ln(F_t + A) = \ln F_t + \ln(1 + \frac{A}{F_t})$ and $\ln(1 + \frac{A}{F_t}) \approx \frac{A}{F_t}$. Now, assuming as before a negligible volatility of the ratio $S_{t+1}/(F_t + A)$, the UIP condition does not hold. The bias is the following:

$$E[\Delta s_{t+1}] - (f_t - s_t) = \frac{A}{F_t} \tag{23}$$

and the precise relationship between the relevant variables follows from equation (13) and is approximated by

$$\Delta s_{t+1} - (f_t - s_t) = \frac{A}{F_t} + \left[\left(\frac{S_{t+1}}{F_t + A}\right) - 1\right] \tag{24}$$

As before, we concentrate on the right hand side of (24). The regression equation is as follows:

$$\left[\left(\frac{S_{t+1}}{F_t + A}\right) - 1\right] = \alpha - A\frac{1}{F_t} + \varepsilon_{t+1} \tag{25}$$

with α and A detecting any inefficiency and/or a risk premium in the foreign exchange market. In practice, however, this model is not tractable given that A is unknown and has an influence in the denominator of the dependent variable. After some algebra (see the mathematical appendix) it can be shown that a tractable version of the regression equation (25) is

$$\left[\left(\frac{S_{t+1}}{F_t}\right) - 1\right] = \alpha + \alpha A\frac{1}{F_t} + \alpha A^2\frac{1}{F_t^2} + \cdots + \alpha A^n\frac{1}{F_t^n} + \eta_{t+1} \tag{26}$$

with $V(\eta_{t+1}) = \frac{\sigma_\varepsilon^2}{(1-\frac{A}{F_t})^2}$, and σ_ε^2 the variance of ε_{t+1}. Since we assume $(\frac{A}{F_t})^2$ and higher orders are negligible, this model can be simplified to:

$$\left[\left(\frac{S_{t+1}}{F_t}\right) - 1\right] = \alpha + \rho\frac{1}{F_t} + \eta_{t+1} \tag{27}$$

where α and $\rho = \alpha A$ measure the degree of market inefficiency. It is well known that although the variance of the error term, η_{t+1}, can be heteroskedastic the parameters estimates from ordinary least squares methods are still unbiased and consistent.

Table 1 Estimated equation $\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \varepsilon_{t+1}$

Currency pair	α	β	R^2	JB	ARCH
Swiss franc/dollar	–0.004 (0.067)	–1.46 (0.02)	0.015	3.75 (0.15)	0.66 (0.57)
Yen/dollar	–0.008 (0.00)	–2.78 (0.00)	0.030	54.10 (0.00)	1.08 (0.50)
Euro/dollar	–0.001 (0.38)	–1.06 (0.15)	0.006	4.51 (0.10)	0.50 (0.68)
Sterling/dollar	0.004 (0.03)	–2.60 (0.00)	0.030	87.60 (0.00)	6.10 (0.00)

p-values are in parentheses

Table 2 Estimated equation $(\frac{S_{t+1}}{S_t} - 1) = \alpha^* + \beta^*(\frac{F_t}{S_t} - 1) + \varepsilon_{t+1}$

Currency pair	α^*	β^*	R^2	JB	ARCH
Swiss franc/dollar	–0.004 (0.11)	–1.48 (0.02)	0.015	7.77 (0.03)	1.05 (0.35)
Yen/dollar	–0.008 (0.00)	–2.82 (0.00)	0.035	37.8 (0.00)	0.74 (0.48)
Euro/dollar	–0.001 (0.00)	–1.14 (0.11)	0.007	9.17 (0.01)	0.80 (0.45)
Sterling/dollar	0.004 (0.02)	–2.60 (0.00)	0.029	110.5 (0.00)	11.9 (0.00)

p-values for are in parentheses

Table 3 Estimated equation $[(\frac{S_{t+1}}{F_t}) - 1] = \alpha + \varepsilon_{t+1}$

Currency pair	α	R^2	JB	ARCH
Swiss franc/dollar	0.002 (0.22)	0.00	0.09 (0.95)	0.15 (0.85)
Yen/dollar	0.001 (0.39)	0.00	13.9 (0.00)	0.07 (0.00)
Euro/dollar	0.001* (0.39)	0.00	4.77 (0.09)	0.83 (0.43)
Sterling/dollar	–0.001* (0.47)	0.00	1.36 (0.50)	0.40 (0.66)

* Significant ARCH effects found when modelling conditional volatility of ε_{t+1}
p-values are in parentheses

Table 4 Estimated equation $[(\frac{S_{t+1}}{F_t}) - 1] = \alpha + \rho \frac{1}{F_t} + \eta_{t+1}$

Currency pair	α	ρ	R^2	JB	ARCH
Swiss franc/dollar	–0.009 (0.37)	0.018 (0.26)	0.003	0.04 (0.97)	0.16 (0.84)
Yen/dollar	–0.005 (0.40)	1.04 (0.28)	0.004	12.1 (0.002)	0.07 (0.93)
Euro/dollar	–0.007* (0.45)	0.016 (0.36)	0.002	3.83 (0.14)	0.42 (0.85)
Sterling/dollar	–0.038* (0.001)	0.023 (0.001)	0.009	1.29 (0.52)	0.01 (0.67)

*Significant ARCH effects found when modelling conditional volatility of ε_{t+1}
p-values for are in parentheses

The conclusions from our proposed tests are the complete reverse of those obtained from the conventional UIP tests. We argue that the negative betas reported by regressions of equation (7) are highly misleading in suggesting the foreign exchange market is inefficient. On the contrary, our results based on regressions (20) and (27) suggest that the foreign exchange market has in fact been efficient for all four bilateral dollar parities under study.

Table 1. Estimates equation $\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \varepsilon_{t+1}$

Currency pair	α	β	R^2	JB	SC-LM	ARCH
Swiss Franc–Dollar	−0.004	−1.46	0.015	3.75	0.63	0.66
	(0.067)	(0.02)		(0.15)	(0.53)	(0.57)
Yen–Dollar	−0.008	−2.78	0.030	54.10	0.35	1.08
	(0.0001)	(0.0)		(0.00)	(0.70)	(0.50)
Euro–Dollar	−0.001	−1.06	0.006	4.51	0.95	0.50
	(0.38)	(0.15)		(0.1)	(0.38)	(0.68)
Sterling–Dollar	0.004	−2.60	0.030	87.6	0.35	6.10
	(0.03)	(0.001)		(0.0)	(0.70)	(0.0)

Note: p values are in parentheses.

Bootstrap Simulation

The method we use for our semi parametric bootstrap algorithm is as follows:

(1) First, we estimate the regression model $\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \varepsilon_{t+1}$ to obtain estimates of $\hat{\alpha}$ and $\hat{\beta}$, we then obtain the residuals (e_{t+1}) of this model as $e_{t+1} = \Delta s_{t+1} - \hat{\alpha} - \hat{\beta}(f_t - s_t)$,

(2) Demean the residuals: $\tilde{e}_{t+1} = e_{t+1} - \bar{e}$ with $\bar{e}$, the sample mean of the sequence of residuals.

(3) For $j = 1,\ldots,B$ with B, the number of bootstrap iterations, we repeat the following:

   a. We generate an independent and identically distributed (*iid*) sequence of observations $h_1^*,\ldots h_n^*$ from a discrete uniform distribution function F taking a value of 1 with probability 0.5, and a value of -1 with probability of 0.5, see Flachaire (2003).

   b. Next, we generate $f_t^* - s_t^* = f_t - s_t$ and $\Delta s_{t+1}^* = \hat{\alpha} + \hat{\beta}(f_t^* - s_t^*) + \tilde{e}_{t+1} h_{t+1}^*$ where the pair $(f_t^* - s_t^*, \tilde{e}_{t+1})$ is the same as in the original regression.

   c. We then estimate the parameters of the regression of Δs_{t+1}^* on $(f_t^* - s_t^*)$ and denote them by $\hat{\alpha}^{*(j)}$ and $\hat{\beta}^{*(j)}$.

(4) We compute the sequence of test statistics $T_{n,\alpha}^{(j)} = (\widehat{\alpha}*(j) - \widehat{\alpha})/se(\widehat{\alpha}*)$ and $T_{n,\beta}^{(j)} = (\widehat{\beta}*(j) - \widehat{\beta})/se(\widehat{\beta}*)$ for $j = 1,\ldots,B$.

(5) Finally, we construct the empirical distribution function of these test statistics:

$$J_n(x,F) = \frac{1}{n}\sum_{j=1}^{n} I(T_{n,i}^{(j)} \leqslant x) \tag{8}$$

with $i = \alpha, \beta$ and $I(T_{n,i}^{(j)} \leqslant x)$ the indicator function taking a value of one if $T_{n,i}^{(j)} \leqslant x$ and zero otherwise.

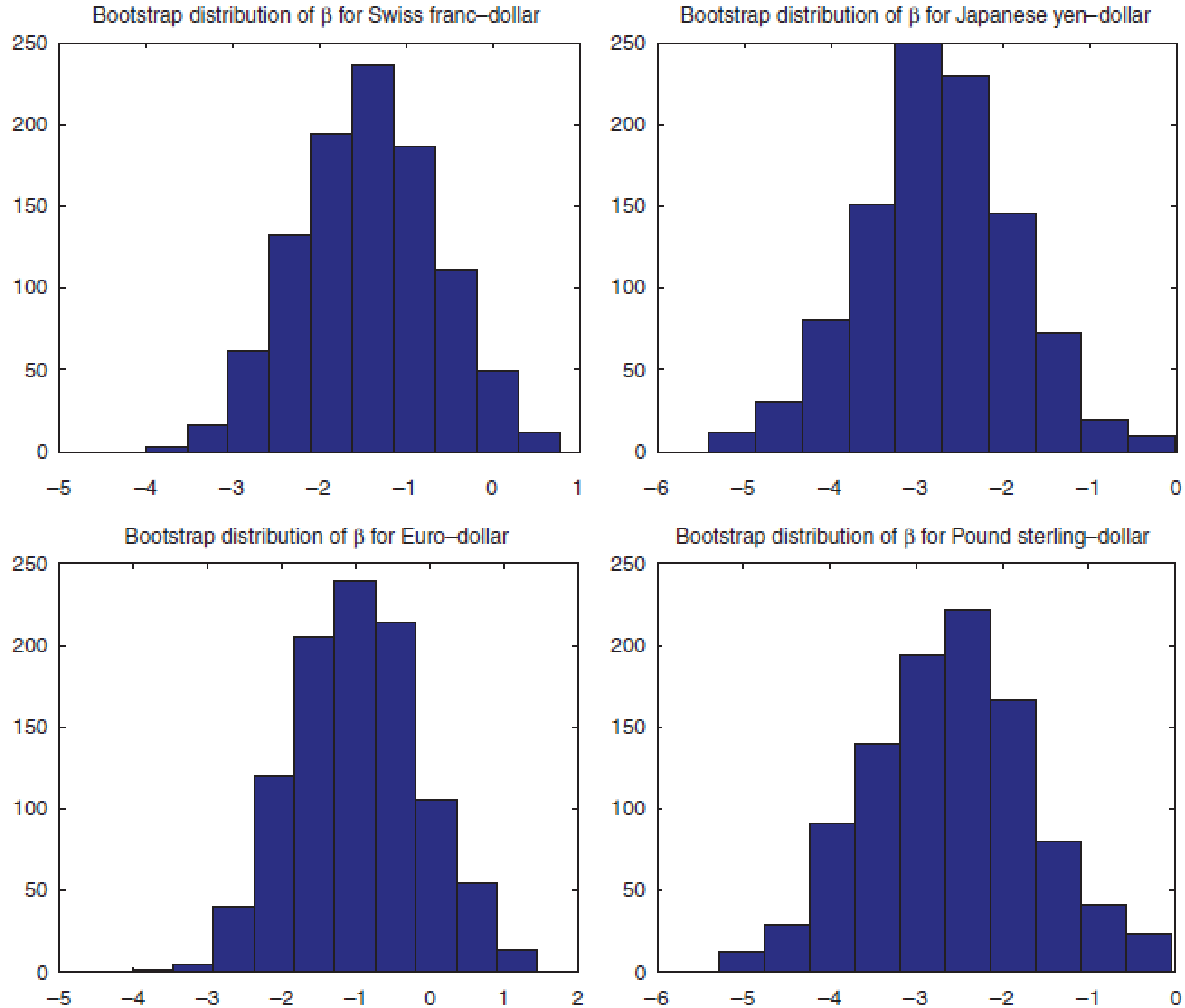


Figure 3. Histograms of the bootstrap estimates of β for four different currency pairs: Swiss Franc–Dollar, Yen–Dollar, Deutchmark/Euro–Dollar and Pound–Dollar for the period November 1978–January 2006. ($n = 327$ observations). $B = 1000$ bootstrap simulations.

Table 2. Bootstrap confidence intervals at 1% for the slope parameter β in equation (7) for the four parities under study

Currency pair	β	Confidence interval β
Swiss Franc–Dollar	−1.46	[−2.99, 0.41]
Yen–Dollar	−2.78	[−5.06, −0.44]
Euro–Dollar	−1.06	[−2.75, 1.01]
Sterling–Dollar	−2.60	[−4.85, −0.02]

$n = 327$, $B = 1000$ bootstrap simulations.

Table 2 also reports the corresponding bootstrap confidence intervals for each currency at a 1% significance level.

Profitability Based Tests of Uncovered Interest Parity

Strategy 1: invest in the foreign currency versus domestic returns

$$R^*_{t+1} - R_{t+1} = \alpha + \varepsilon_{t+1} \qquad (19)$$

Market efficiency boils down to see if $\alpha = 0$ and the error term is a white noise. Note the regression is balanced if $R^*_{t+1} - R_{t+1}$ and the error term are stationary. If this is the case and the error term is a white noise, Ordinary Least Squares (OLS) estimators provide consistent and efficient estimates of the parameters. Conversely, if $\alpha \neq 0$, there exists either a risk premium and/or market inefficiency between the currency pair.

Table 3. Estimated equation $R^*_{t+1} - R_{t+1} = \alpha + \varepsilon_{t+1}$

Currency pair	α	R^2	JB	SC-LM	ARCH
Swiss Franc–Dollar	−0.001	0	2.36	1.90	0.47
	(0.58)		(0.30)	(0.14)	(0.62)
Yen–Dollar	−0.0004	0	38.5	0.35	1.40
	(0.79)		(0.0)	(0.05)	(0.24)
Euro–Dollar	−0.0001	0	1.25	2.20	0.37
	(0.94)		(0.53)	(0.11)	(0.76)
Sterling–Dollar	0.002	0	50.9	1.22	0.98
	(0.27)		(0.0)	(0.29)	(0.40)

Note: *p* values are in parentheses.

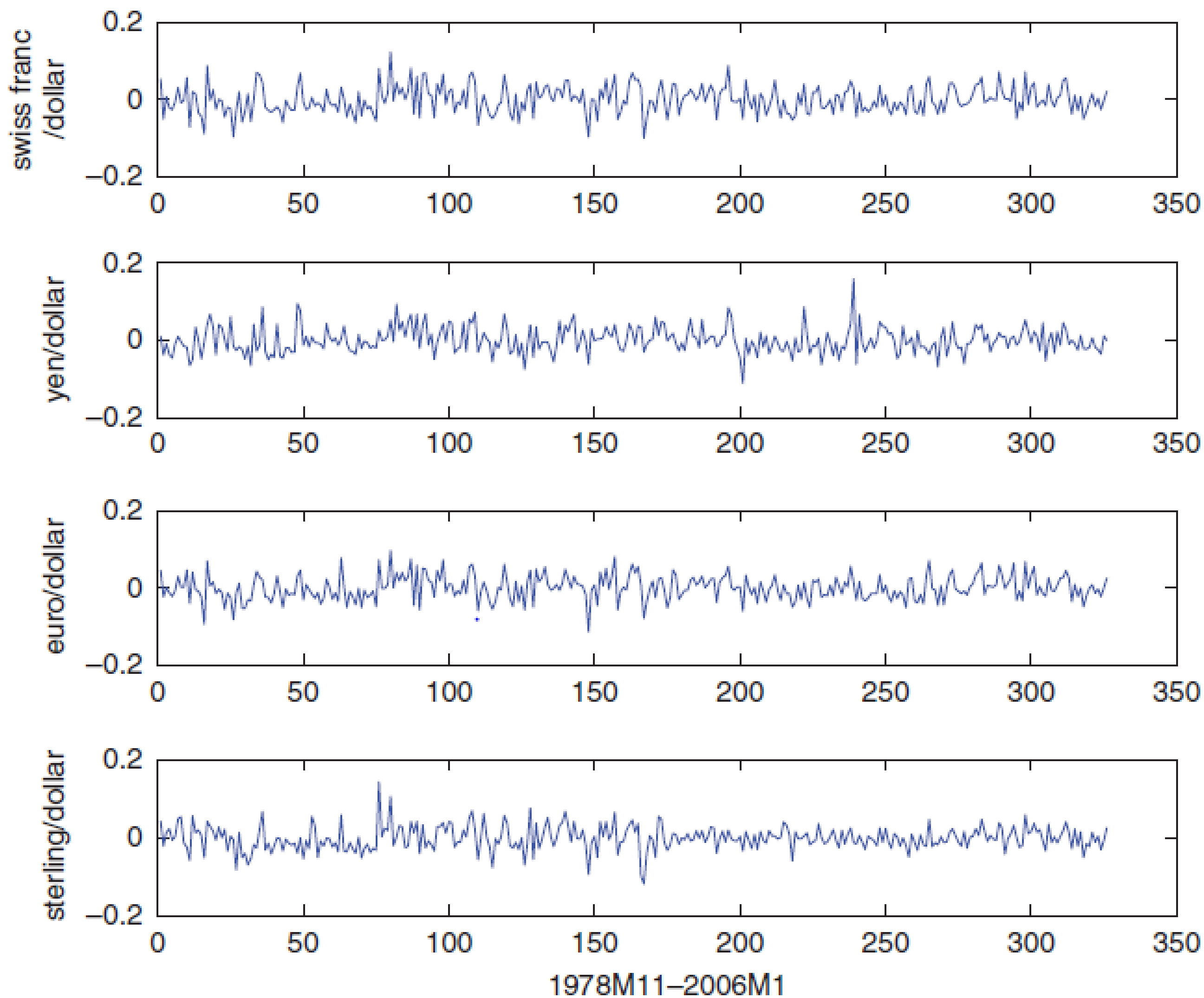


Figure 4. Residual sequences of the regression equation $R^*_{t+1} - R_{t+1} = \alpha + \varepsilon_{t+1}$ for four different currency pairs: Swiss Franc–Dollar, Yen–Dollar, Deutchmark/Euro–Dollar and Pound–Dollar for the period November 1978–January 2006. ($n = 327$ observations).

Profitability Based Tests of Uncovered Interest Parity

Strategy 2: invest in the currency that yielded the highest return in the previous period versus domestic returns

$$\mathrm{RHR}_{t-1} - R_{t+1} = \alpha + \varepsilon_{t+1} \quad (20)$$

where RHR_{t+1} is the actual dollar return from being in the previous period's highest return currency.

The highest return or momentum strategy consists of investing in each period in the currency that yielded the highest return (interest and exchange rate movement) in the previous period. The investment rule is as follows, if $R^*_{t-1} - R_{t-1} < 0$, we invest for the current period in the US market as this yielded the highest return in the previous period. Conversely, if $R^*_{t-1} - R_{t-1} > 0$, then the investor places their capital in the foreign currency for the current period. This rule is equivalent to investing in the US dollar if $S_t > F_{t-1}$ and in the foreign currency otherwise. If the market is efficient this momentum-based strategy should not lead to any excess return compared with holding the capital in domestic currency. The appropriate regression test of the strategy given by equation (20), which compares the realized returns of this momentum-based strategy against just holding the capital to earn the domestic interest rate.

Table 4. Estimated equation $RHR_{t+1} - R_{t+1} = \alpha + \varepsilon_{t+1}$

Currency pair	α	R^2	JB	SC-LM	ARCH
Swiss franc–Dollar	0.001	0	159.5	0.68	0.14
	(0.41)		(0.0)	(0.50)	(0.93)
Yen–Dollar	0.002	0	1058	1.03	3.29
	(0.12)		(0.0)	(0.35)	(0.02)
Euro–Dollar	0.002	0	112.7	0.44	7.4
	(0.05)		(0.0)	(0.64)	(0.96)
Sterling–Dollar	0.002	0	186	1.75	0.29
	(0.057)		(0.0)	(0.17)	(0.83)

Note: p values are in parentheses.

Profitability Based Tests of Uncovered Interest Parity

Strategy 3: invest in the high interest rate currency versus returns in the low interest rate currency

$$RH_{t+1} - RL_{t+1} = \alpha + \varepsilon_{t+1}$$

where $E[RH_{t+1}]$ is the expected dollar return from being in the high-interest rate currency and $E[RL_{t+1}]$ is the expected dollar return from being in the low-interest rate currency. The economic interpretation of this condition in this context is that an investor does not obtain excess profits by placing their capital in the high- or low-interest rate currency.

Table 5. Estimated equation $RH_{t+1} - RL_{t+1} = \alpha + \varepsilon_{t+1}$

Currency pair	α	R^2	JB	SC-LM	ARCH
Swiss Franc–Dollar	0.002	0	12.1	1.49	0.38
	(0.21)		(0.002)	(0.22)	(0.76)
Yen–Dollar	0.003	0	56.03	2.00	1.61
	(0.086)		(0.0)	(0.13)	(0.18)
Euro–Dollar	0.004	0	10.8	1.22	0.21
	(0.02)		(0.01)	(0.29)	(0.89)
Sterling–Dollar	0.005	0	56.5	1.00	1.49
	(0.0007)		(0.0)	(0.36)	(0.21)

Note: p values are in parentheses.

Profitability Based Tests of Uncovered Interest Parity

Strategy 4: buy the currency at a forward discount in the forward market versus domestic returns

$$\mathrm{RF}_{t+1} - R_{t+1} = \alpha + \varepsilon_{t+1}$$

The forward strategy involves the investor going long with all their capital plus the relevant US interest rate during the holding period in the currency that is at a forward discount in the market. The idea of this strategy is that if one takes the negative beta coefficient or low-positive beta coefficients from conventional regression analyses of UIP, the currency at a forward discount (*i.e.* the high interest rate currency) is actually likely to appreciate or at least not depreciate as much as the forward discount indicates. If this happens, then the realized exchange rate S_{t+1} will be less than the forward rate F_t and the investor will realize an excess return from the forward purchase of the high-interest rate currency compared with just holding their capital at the domestic interest rate. This profitability strategy is the same as Strategy 3 when the US interest rate is the highest but differs when the US interest rate is below the foreign interest rate.

Table 6. Estimated equation $RF_{t+1} - R_{t+1} = \alpha + \varepsilon_{t+1}$

Currency pair	α	R^2	JB	SC-LM	ARCH
Swiss Franc–Dollar	0.002	0	12.05	1.57	0.55
	(0.22)		(0.002)	(0.21)	(0.57)
Yen–Dollar	0.003	0	56.03	2.00	1.61
	(0.086)		(0.0)	(0.13)	(0.18)
Euro–Dollar	0.004	0	10.2	1.25	0.21
	(0.02)		(0.005)	(0.28)	(0.88)
Sterling–Dollar	0.006	0	55.4	0.34	1.49
	(0.0003)		(0.0)	(0.71)	(0.18)

Note: *p* values are in parentheses.

Summary

The results of our profitability-based tests show that despite a decisive rejection of UIP using the conventional regression-based approach, it, nonetheless, seems that the foreign exchange market is efficient for at least two of the four parities studied which pass all four of our tests, namely, the Swiss Franc–Dollar and Yen–Dollar parities.

In the case of the Euro–Dollar and Pound–Dollar parities the evidence is somewhat mixed as they pass our first two tests but not the latter two.

Summary

The profitability tests that we have proposed have a key advantage in distinguishing between bilateral parities where the foreign exchange market can said to be efficient and parities where it may not of been.

Our results based on excess returns also seem to have better statistical properties than the conventional UIP regression results which are unreliable due to different statistical pitfalls such as the omission of relevant variables, the large difference in volatility between the dependent variable and the explanatory variable and the presence of conditional heteroskedasticity in the data.

Summary

Our view is that the conventional rejections of UIP and with it market efficiency are primarily a statistical phenomenon and are at best only indirect tests of the efficient market hypothesis. It is clear that based on the criterion of profitability the Swiss Franc–Dollar and Yen–Dollar foreign exchange markets are efficient in stark contrast to the conclusion reached with the UIP based regression test results.

Conclusion/ Takeaway !

The key advantage of our proposed tests is that they constitute a more direct test of the efficient market hypothesis and accord with the idea of an efficient market being one in which is difficult for market participants to make excess returns from pursuing rather simplistic trading strategies. The results of suggest that foreign exchange market is far more efficient than the existing literature on UIP has given it credit for.

Our results suggest that it may well be the case that a substantial body of economic literature that has rejected the UIP condition due to either the market irrationality or the existence of time varying risk-premia or a combination of the two also needs to be re-examined.